Endogenous Authority and Enforcement in Public Goods Games*

Wooyoung Lim† Jipeng Zhang‡

Abstract

This paper investigates theoretically and experimentally the social benefits and cost to have an endogenous punishment-enforcing authority in public goods game. An authority is chosen among members of a society via an imperfectly discriminating contest prior to a public goods game. Once chosen the authority has a large degree of discretion to inflict punishment. Our theoretical result shows that an efficiency gain from having the endogenous authority always comes with a social cost from competing for being the authority. The larger the society is, however, the bigger the efficiency gain and the smaller the rent dissipation. The completely efficient outcome can be approximated as the size of society tends to infinity. The experimental results confirm that the presence of endogenous authority for a given group size increases the public goods contributions and the efficiency gain is significantly bigger in a larger group.

Keywords: Endogenous Authority; Punishment; Public Goods; Group Size

JEL classification: C72; C92; D82; D83

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1 Introduction

The free-rider problem in public goods provision is one of the classical topics in economic theory. Recent studies largely focused on how to design mechanisms or institutions that can improve cooperation. It is well understood that not only centralized punishments but also decentralized peer-to-peer sanction schemes can reduce the free-riding behavior in the public goods environment.\(^1\) The literature also investigates how institutions and sanction schemes are generated and their impacts on efficiency and social welfare (Kosfeld et al. (2009); Dal Bó et al. (2010)).

In most previous studies, no clear distinction has been made between the endogenous formation of institutions/authorities and the endogenous choice of punishment scheme, except Yamagishi (1986) and O’Gorman et al. (2008) who study how endogenously chosen punishment levels by a central sanctioning authority can improve contributions. In considering the endogenous choice of informal (e.g., Ertan et al., 2009; Sutter et al., 2010) and formal sanctions (e.g., Tyran and Feld, 2006; Kroll et al., 2007), the endogenous formation of authorities essentially implies that the choice of punishment scheme is made, mostly via a voting mechanism, and the chosen punishment scheme becomes a binding constraint in the public goods provision. That is, in these studies, the authority is defined as a device that mechanically and passively implements the chosen sanction scheme.

In this paper, we consider an authority that has a large degree of freedom to decide who to punish and the punishment severity \textit{ex post}, and explore endogenous formation of such authorities in the standard public goods framework. This is mainly motivated by the fact that many institutions or authorities in our society are often created, funded, and operated \textit{endogenously} by all or a subset of the group members, and these endogenously formed authorities have power and certain degrees of freedom to enforce rules and punishment. Examples of endogenous authority and enforcement include regulation and law enforcement agents, and discipline committees of political and professional organizations. The objective of this paper is to evaluate the efficiency and welfare of public goods provision when we consider both endogenous authority formation and endogenous punishment enforcement.

Although the existence of authority can lead to more public goods contribution, endogenizing the punishment-enforcing authority entails social costs. One immediate cost is that the authority selected might have no incentives to contribute to public goods since the possibility of being punished as an authority is small or even zero. This in turn creates a rent the members of society pursue and results in a socially wasteful rent dissipation. The real costs the society has to bear would be even higher in reality, given that the authority may have positive intrinsic value to people (Fehr et al., 2013), the authority may abuse his/her power (Muthukrishna et al., 2017), and/or members of the society may not fully comply with the authority due to a legitimacy concern (Dickson et al., 2017). Overall, when

\(^1\)Hung and Plott (2001) and Levine and Palfrey (2007) summarizes part of the large literature on the determinants of institution formation and efficiency. The impact of rewards and punishments in public goods provision is studied by, for example, Ostrom et al. (1992), Fehr and Gächter (2000), Andreoni et al. (2003), and Fudenberg and Pathak (2010).
the formation of the punishment-enforcing body is endogenous, the extent to which the endogenous authority exercises its enforcement power and how much it can improve efficiency is an empirical question. Hence, a theory-based experimental study can be valuable.

Theoretically, we consider a three-stage game among $N$ risk-neutral players. In the first stage, they participate in an imperfectly discriminating contest à la Tullock (2001) and make irreversible efforts to win the contest.\(^2\) Contest is an appropriate way to capture the costly competition for power among players who possibly seek rent after being selected as the authority.\(^3\) In the second stage, all members of the society including the winner of the first stage contest participate in a linear public goods (LPG) game. In the third stage, the levels of individual public goods contribution are revealed only to the winner of the contest (the authority) who will decide who to punish and the punishment severity.\(^4\)

In this game there is an equilibrium that has many desirable properties when certain conditions are satisfied. For example, the efficiency gain is bigger for a larger society, and the fully efficient outcome can be approximated as the size of the society goes to infinity. But the positive efficiency gain from the endogenous authority is always accompanied by the positive social cost of endogenizing the authority. In equilibrium the total contest spending is always positive and the authority contributes nothing to the public goods. For a larger society, the equilibrium predicts that the total contest spending decreases and thus the social cost becomes negligible in theory.

We design an experiment to explore the social benefits and costs of having the endogenous authority in groups of different sizes as described in our model. The main theoretical predictions are consistent with the experimental results.\(^5\) With an endogenous authority, the efficiency gain is significantly higher in the larger group than that in the smaller group. The average contribution to the public goods is also higher in the larger group treatment. A larger group size reduces the individual contest spending, but leads to more severe punishment. Authorities contribute a moderate amount to the public goods, even without the fear of being punished for free-riding.

This paper contributes to the literature on the different sanctioning schemes in promoting cooperation in public goods games. Both peer-to-peer punishment and formal sanction can increase

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\(^2\)Morgan (2000) is the first to consider a contest model in the provision of public goods. He studies a lottery contest whose proceeds will be used to fund a public good.

\(^3\)It is possible that the authority is chosen via some other, possibly more democratic, mechanisms such as voting. In this case, our game can be interpreted as a reduced-form version of the political rent-seeking game in which a political rent-seeking contest is held prior to the voting. Spindler (1990) considers constitutional design for a rent-seeking society and investigates a choice of voting rule. For more details about political rent-seeking, see Krueger (1974) and Hillman and Riley (1989).

\(^4\)In the one-shot game context, whether or not the punished person and the punishment amount are revealed to the society does not matter. Although we do not reveal such information to public in our experimental implementation due to the potential learning issue, it would be interesting to consider the case in which the amount of punishment becomes public information and see how people react.

\(^5\)There is an issue of multiple equilibria. The theoretical predictions are based on a particular equilibrium outcome in which the authority chooses the lowest contributor. We find that the observed behavior of the authority in our data is qualitatively consistent with the equilibrium outcome.
contributions to public goods.\footnote{Examples of peer-to-peer punishment are Fehr and Gachter (2000), Denant-Boemont et al. (2007), Nikiforakis (2008), Fudenberg and Pathak (2010), Przepiorka and Diekmann (2013), and Gross et al. (2016). For informal sanctions, see Falkinger et al. (2000), Boyd et al. (2010), Sigmund et al. (2010), and Fehr and Williams (2018). Diekmann and Przepiorka (2015) discusses how more centralized forms of punishment endogenously emerge.} Recent studies have investigated the selection of formal and informal sanctions through voting (e.g., Kosfeld et al., 2009; Sutter et al., 2010; Kamei, 2016; Traulsen et al., 2012; Markussen et al., 2013; Nicklisch et al., 2016) and found that subjects are willing to face punishment by peers or a third party. Andreoni and Gee (2012) demonstrate a less costly and more efficient mechanism that allows members to hire a “gun” from outside to enforce a given punishment. Unlike Andreoni and Gee (2012), we assume that the gun is chosen among members of the society endogenously and once chosen, the gun has a large discretion to use a variety of punishments or to use none at all. Different from this literature, our paper is the first to combine an endogenously chosen authority with endogenously chosen punishments in a theoretical analysis.\footnote{However, the endogenous choice of punishment in our paper is not exactly the same as that in the papers mentioned. Previous literature considers a collective choice of punishment whereas in our paper we consider a process to let an individual emerges endogenously as an authority and let the individual choose punishment.}

We also contribute to the literature on the relationship between public goods contribution and group size. Isaac and Walker (1988) find that public goods contributions tend to rise with group size (see also Isaac et al. (1994)). Carpenter (2007b) examines how punishment is affected by group size and finds that large groups contribute at rates no lower than small groups because punishment does not fall appreciably in large groups. However, hindrances to monitoring do reduce the provision of the public good. In our paper, people contribute more in a larger group, but through a different mechanism. In our theory, each individual wastes less resource in competing to be the authority in a larger group while the punishment power of the authority is bigger in a large group.

Our paper is also related to the literature on the endogenous institutional choice and its impact on social welfare.\footnote{Earlier literature has studied exogenously imposed institutions (e.g., Ostrom et al., 1992; Chen and Plott, 1996; Masclet et al., 2003; Anderson and Putterman, 2006; Carpenter, 2007a,b).} Ertan et al. (2009) and Sutter et al. (2010) find that punishments and rewards in public goods games have a greater impact on behavior when they are allowed democratically. Levine (1990), Bonin et al. (1993) and Black and Lynch (2001) find that economic agents are more cooperative or productive when they are allowed to decide the rules with greater democratic participation. Dal Bó et al. (2010) further confirm the importance of democratic decision-making to the efficiency of institutions. Most of these previous studies on institutions, however, abstract away from possible enforcement problems that might arise when both the institution and the enforcement of punishment are endogenous.
2 Model

We derive and test our theory in the context of a linear public goods (LPG) game that is a perfect setting to create the essential trade-off between the cost generated from the competition for authority and the benefit of having a more cooperative outcome in the public goods game. Individually, the trade-off is between the individual spending to become an authority and the benefit from not being punished for free-riding behavior, as the authority in our model has discretion to choose whom to punish and the punishment level.

2.1 The Endogenous Authority

Consider a society with \(N \geq 2\) risk-neutral players who participate in the following game.

Stage 1. Pre-LPG-game contest. Each individual player \(i \in I = \{1, 2, \ldots, N\}\) is endowed with \(\omega > 0\) and invests \(x_i \in [0, \omega]\) independently and simultaneously to win the pre-LPG-game contest. The winner of the contest will become the authority in the punishment stage (Stage 3) and earns an exclusive right to decide who to punish and the punishment severity, subject to certain constraints. Let \(x = (x_1, \ldots, x_N)\). Player \(i\)'s winning probability in the pre-LPG-game contest is determined by the following contest success function:

\[
P_i(x) = \begin{cases} 
0 & \text{if } \sum_{j=1}^{N} x_j = 0 \\
\frac{x_i}{\sum_{j=1}^{N} x_j} & \text{otherwise}
\end{cases}
\]  

Stage 2. LPG Game. Each individual \(i\), no matter whether he/she is the winner of the pre-LPG-game contest, allocates \(y_i\) (\(0 \leq y_i \leq \omega - x_i\)) to the public goods and consumes the rest \(\omega - x_i - y_i\). Each receives one unit of utility for each unit of \(\omega\) kept, and receives \(\alpha\) unit of utility for each unit allocated to the public goods by everyone, where \(0 \leq \alpha < 1\) and \(N\alpha > 1\).\(^9\) So, player \(i\)'s interim payoff is

\[
\pi_i = \omega - x_i - y_i + \alpha \sum_{j=1}^{N} y_j.
\]  

Stage 3. Punishment Stage. The profile of individual contributions, denoted by \(y = (y_1, \ldots, y_N) \in Y \subset \mathbb{R}^N\), is revealed only to the winner of the pre-LPG-game contest, i.e. the authority. The authority decides who to punish and the severity of punishment. Formally, the authority’s strategy is a mapping from \(Y\) to \(\mathbb{R}^N\) that specifies the severity of punishment for each player. Let \(s = (s_1, \ldots, s_N) \in \mathbb{R}^N\). Then \(s\) can be, for example, \((0,0,\ldots,0,1)\), meaning that only the \(N\)th person is punished with severity \(1\).

The authority’s decision is constrained by the following two conditions: 1) the authority chooses

\(^9\)These assumptions are to ensure that the unique outcome of the sub-game perfect Nash equilibrium of the stage game (LPG) is complete free riding, i.e. \(y_i = 0\) for every \(i \in I\).
only one member to punish, i.e. there exists at most one \( i \in I \) such that \( s_i > 0 \), and 2) \( \Sigma_{i=1}^{N} s_i \leq \beta \cdot \Sigma_{i=1}^{N} x_i \)

where \( \beta > 0 \) is exogenously given, which implies that the total contest expenditure serves as a constraint on the maximum severity of punishment.\(^\text{10}\) The first constraint that the authority has to choose only one member to punish has empirical relevance and we discuss this issue carefully in Section 5. The second assumption may not look entirely reasonable, however, without such an endogenous constraint, the punishment mechanism should become unrealistically powerful.

Given the decision \( s \) made by the authority, the final payoff of player \( i \) becomes

\[
\Pi_i = \begin{cases} 
\pi_i - s_i = \omega - x_i - y_i + \alpha \Sigma_{j=1}^{N} y_j - s_i & \text{if } \pi_i \geq s_i \\
0 & \text{otherwise.}
\end{cases}
\]  

\( (2.3) \)

Note that punishment is, beyond paying for the contest, not individually costly for the selected authority. Instead, the cost is shared across group members via their contribution to the first stage Pre-LPG-game contest. This feature of our punishment mechanism is an important deviation from unregulated peer punishment (e.g., Fehr and Gächter (2000)) and is closer to pool punishment mechanisms (e.g., Ozono et al. (2016)).\(^\text{11}\)

### 2.2 Equilibrium and Welfare Analysis

This section shows that there exists a subgame perfect equilibrium (hereafter equilibrium) with several desirable properties. For each \( i \in I \), we define \( y_i^z \) as the smallest element of the vector \( y_i = (y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_N) \), that is \( y_z \leq y_j \) for all \( j \in I \setminus \{i\} \), and we denote the set of other players with the smallest contributions by \( L_i = \{ j \in I \setminus \{i\} \mid y_j = y_i^z \} \).

Consider the following strategy profile. In the third (punishment) stage, the authority, denoted by \( \hat{a} \), punishes members of the society with severity \( s_j = \hat{s} \leq \beta \Sigma_{k=1}^{N} x_k \) for \( j \in \hat{L} \) if \( \hat{L} \) is singleton. Otherwise \( s_j = 0 \) for any \( j \in I \). In other words, the single lowest contributor, whenever exists, is punished with severity \( \hat{s} \). We are interested in a symmetric equilibrium in which every individual contributor \( i \) invests the same amount \( x_i \) to win the contest. It is straightforward to see that all equilibria must be symmetric in terms of the contest spending, if the lowest contributor is punished by the authority.\(^\text{12}\) In the second (public goods provision) stage a player \( i \) contributes nothing if he/she is

\(^\text{10}\)Our second constraint for the authority’s decision is related to Ozono et al. (2016) who consider a public good provision with a leader support system in which an authority (exogenously chosen) can freely punish other group members using capital pooled through the support of group members. The main difference is that the individual spending in contest is not just to provide capital for punishment in our mechanism but also to compete for being the authority.

\(^\text{11}\)Moreover, it is not possible for the authority to abuse his or her power. If the budget for punishment could, for example, be siphoned off as income, then our equilibrium construction would not work. For more discussion, see e.g., Muthukrishna et al. (2017).

\(^\text{12}\)At the end of this section, we will further discuss the validity of the assumption that the lowest contributor among non-authority members is punished.
the winner of the pre-LPG-game contest; otherwise he/she contributes all the remaining endowment to the public goods, i.e. \( y_i = \omega - x_i \).

Notice that, given this strategy profile, everyone contributes the same amount except the authority. As a result, no one will be punished on the equilibrium path. However, the no-punishment outcome is a phenomenon only at the exact equilibrium fixed point. Considering any epsilon mistake or tremble, it is more sensible to expect that punishment, according to the equilibrium strategy, takes place.

The expected utility of each individual \( i \) in the first stage becomes

\[
EU_i = \frac{x_i}{\sum_{j=1}^{N} x_j} \left( (\omega - x_i - y_i^w + \alpha \sum_{j=1}^{N} y_j) + (1 - \frac{x_i}{\sum_{j=1}^{N} x_j}) (\omega - x_i - y_i^l + \alpha \sum_{j=1}^{N} y_j - s_i) \right)
\]

where \( y_i^w \) and \( y_i^l \) denote the individual’s public good contribution conditional on winning and losing the contest, respectively. By backward induction, we have \( y_i^w = 0 \), \( y_i^l = \omega - x_i \) and \( s_i = 0 \). Then the expected utility of individual \( i \) becomes

\[
EU_i = \frac{x_i}{\sum_{j=1}^{N} x_j} \left( (\omega - x_i + \alpha \sum_{j=1}^{N} y_j) + \alpha (1 - \frac{x_i}{\sum_{j=1}^{N} x_j}) (\omega - x_i + \sum_{j \neq i} y_j) \right)
= \left( 1 - \frac{1}{N} \right) \left( (\omega - x_i) \frac{x_i}{\sum_{j=1}^{N} x_j} - \frac{\alpha}{1 - \alpha} x_i \right) + \alpha \omega.
\]

Hence, each individual \( i \) chooses the contest expenditure \( x_i \) to solve:

\[
\max_{x_i \in [0, \omega]} \left( (\omega - x_i) \frac{x_i}{\sum_{j=1}^{N} x_j} - \frac{\alpha}{1 - \alpha} x_i \right). \tag{2.4}
\]

From the first-order condition,\(^{13}\) we get

\[
- \frac{x_i}{\sum_{j=1}^{N} x_j} + (\omega - x_i) \frac{\sum_{j \neq i} x_j}{(\sum_{j=1}^{N} x_j)^2} = \frac{\alpha}{1 - \alpha}.
\]

In a symmetric equilibrium, \( x_j = x_i \) for all \( j \), and the above first-order condition boils down to

\[
- \frac{1}{N} + (\omega - x_i) \frac{N - 1}{(Nx_i)^2} = \frac{\alpha}{1 - \alpha}.
\]

After rearranging, we have

\[
x_i = \omega \cdot \left( \frac{N - 1}{\alpha (1 - \alpha) N^2 + 2N - 1} \right) \equiv x^*; \tag{2.5}
\]

We need to check that an individual player \( i \) has no incentive to deviate in the public goods

\(^{13}\) The second-order condition is also satisfied.
provision stage. Given all the other players follow the equilibrium strategy, any deviation toward a smaller amount of contribution will result in punishment of severity $\bar{s}$. Thus, it suffices to consider the most extreme deviation, $y_i^d = 0$. Clearly the gain from this deviation is $\omega - x^*$. The loss from this deviation consists of two components. First, the total public goods provision is reduced by $\omega - x^*$ which generates a utility loss $\alpha(\omega - x^*)$. Second, the punishment of severity $\bar{s}$ will be handed down in the third stage. The deviation should be unprofitable, i.e.

$$x^* \geq \omega - \frac{\bar{s}}{1 - \alpha}. \quad (2.6)$$

From (2.5) and (2.6), we have

$$\bar{s} \geq \frac{\omega(1 - \alpha)N((\frac{N}{1-\alpha})N + 1)}{N^2 + 2N - 1} \equiv \bar{s}. \quad (2.7)$$

Such an $\bar{s}$ exists only when the right-hand side of (2.7) is (weakly) smaller than the maximum possible punishment $\beta N x^* \equiv \bar{s}$. Thus, we have

$$N \geq \frac{\beta + (1 - \alpha)^2}{\beta - \alpha(1 - \alpha)} \quad (2.8)$$

when $\beta > \alpha(1 - \alpha)$.

**Proposition 1.** For any $\alpha \in (0, 1)$ and $\beta > \alpha(1 - \alpha)$, there exists $N^*$ such that for any $N > N^*$ there exists an equilibrium in which for any $i \in I$, $x_i = x^*$, and for any $j$ who is not the authority, $y_j = \omega - x^* \equiv y^*$. Moreover, $x^*$ and $y^*$ converge to 0 and $\omega$ respectively, when $N$ tends to infinity.

**Proof.** From the condition (2.5), we get the following necessary condition:

$$\frac{\omega N(N - 1)}{(\frac{N}{1-\alpha})N^2 + 2N - 1} > 0, \quad (2.9)$$

which holds for any $N > 1$. The convergence is an immediate result of (2.5).\hfill $\square$

The above result says that, as long as the authority has enough power to hand down punishment ($\beta > \alpha(1 - \alpha)$), the proposed mechanism can achieve the socially desired outcome in a large society ($N > N^*$). Note that, as demonstrated in (2.8), a continuum of $\bar{s}$ in $[\frac{\omega(1-\alpha)N((\frac{N}{1-\alpha})N + 1)}{(\frac{N}{1-\alpha})N^2 + 2N - 1}, \frac{\omega \beta N(N - 1)}{(\frac{N}{1-\alpha})N^2 + 2N - 1}] = [\bar{s}, \bar{s}]$ can achieve the equilibrium outcome.

The intuition behind this result is the following. First, as winning in the pre-LPG-game contest is probabilistic whereas punishment in the last stage is deterministic, an individual has no incentive to invest too much in the contest stage. Moreover, the cost of investing more in the contest stage is higher than that of investing more in the standard rent-seeking contest since in the current game the value of winning, $(\frac{1-\alpha}{\alpha})(\omega - x^*)$, decreases as $x^*$ increases. Consistent with the standard results of
comparative statics in the rent-seeking contest literature, however, as the size of the society increases, the individual contest expenditure decreases whereas the total contest expenditure increases.

Our intuition is confirmed by the following comparative statics. From equation (2.5), the individual contest spending \( x^* \) is strictly decreasing in \( N \). The total contest spending \( Nx^* = \frac{\omega N(N-1)}{(1-\alpha)N^2 + 2N - 1} \) is strictly increasing in \( N \) but converges to \( \left( \frac{1-\alpha}{\alpha} \right) \omega \) from below. As a result, both the individual public goods provision \( y^* \) and the total public goods provision \((N - 1)y^* \) are strictly increasing in \( N \).

The above result highlights that the benefit of endogenous authority is always accompanied by the cost of endogenizing the authority. The benefit of having an endogenous authority can be measured by the (percentage) efficiency gain as follows:

\[
B(\%) = 100 \times \frac{(N - 1)y^*}{N\omega} = 100 \times \frac{(N - 1)((\frac{\alpha}{1-\alpha})N + 1)}{(\frac{\alpha}{1-\alpha})N^2 + 2N - 1}.
\]

The cost consists of two components. First, after being selected, the authority does not contribute anything to the public good. Second, the society as a whole spends \( Nx^* \) to pursue the rent. The (percentage) efficiency loss from the social cost of endogenizing the authority, denoted by \( C \), is defined as follows:

\[
C(\%) = 100 \times \frac{(\omega - x^*) + Nx^*}{N\omega} = 100 \times \frac{(\frac{1}{1-\alpha})N}{(\frac{\alpha}{1-\alpha})N^2 + 2N - 1}.
\]

Note that \( C = 100 - B \), which is due to the fact that every non-authority individual contributes the full amount he/she has after pre-LPG-game contest to public goods in equilibrium. Figure 1 demonstrates that the percentage efficiency loss converges to 0% as \( N \) approaches infinity, despite the fact that the efficiency loss itself is always positive.

![Figure 1: Social cost measured by C (%) with \( \alpha = \frac{1}{2} \)](image)

Several remarks mostly on the multiplicity of equilibria are in order. First, we focus on the symmetric equilibrium in which every individual contributes the same amount in the second stage contest. This assumption is without loss of generality as long as we consider an equilibrium in which the lowest contributor is punished. Thus, the important issue remaining is whether or not it is natural...
to focus on the equilibrium outcome in which the authority applies the deterministic punishment rule to the lowest contributor.\textsuperscript{14} Second, one may find that it is too strong to assume that the authority fully exercises his punishment power \textit{ex-post}. As one can see in equation (2.8), however, it is obvious that a broad class of rules with less extreme punishment can lead to a similar outcome with a slower convergence. Another extreme type of equilibrium of the model for a large range of parameters is the one in which no one contributes positively to the contest and thus no authority is formed. When no one invests anything in the contest stage, the value of winning the contest is zero and, as a result, no one has an incentive to deviate and finance himself/herself to punish others. In this sense, the emergence of an authority is genuinely endogenous. An authority may or may not be formed depending on different equilibria. Our experimental investigations in the next section will offer useful insights on these issues.

3 Experimental Design and Procedures

3.1 Treatments and Hypotheses

Under the theoretical guidance, we set $\omega = 50$, $\alpha = 0.75$, and $\beta = 3$ and consider two different group sizes, $N = 4$ and 8. We set $\beta = 3$, which gives the authority only moderate power to hand down punishment.\textsuperscript{15}

Table 1 presents our treatments and experimental conditions. We adopt a $2 \times 2 \times 2$ design. The major treatment variables correspond to the group size, and the order of experimental conditions with and without the pre-LPG-game contest for the endogenous authority. We name each condition based on the group size and whether or not it includes endogenous authority and punishment. For example, Condition \textit{NoA4} stands for \textit{No Authority} with the group size 4. As shown in Table 1, there are four treatments, each of which involves two conditions with the same group size. In each session, subjects participate in the linear public goods games with and without endogenous authority in each of the first and second 15 rounds in the specified order. For example, in Treatment 1, Condition \textit{NoA4} is adopted in the first 15 rounds and Condition \textit{A4} is adopted in the second 15 rounds.

The design allows us to explore whether our finding would be robust to different orders. Theoretically, the order of treatments does not change the major predictions of the game, including the public goods contribution, income, and contest spending. We are interested to see whether the endogenous authority is affected by history or subjects’ experience. Conditions with and without the endogenous authority differ with respect to the complexity of games, the amount of time subjects expect to spend,\textsuperscript{14}\textsuperscript{15}

\textsuperscript{14}This is indeed the case in our experimental data: the authorities chose the \textit{lowest} contributor to punish 201 times out of 240 (83.8\%) in the treatments with group size of 4 and 189 times out of 240 (78.8\%) in the treatments with group size of 8. For more details, see the experimental result section, in particular, Section 4.2.

\textsuperscript{15}It has been found that radical punishment is not necessary to achieve a significant improvement in public goods provision (Andreoni and Gee, 2012).
expected income, and so on. We control for such factors by having both conditions with and without the endogenous authority in each treatment.

Our hypotheses are based on the theoretical predictions presented in Table 2. We first investigate the welfare improvement from having an endogenous authority by comparing the contributions to the public goods in the first and the second 15 rounds for a fixed group size and a fixed order.

**Hypothesis 1.** For a given order of conditions, the individual contribution to the public goods in Condition \( A4 \) (\( A8 \)) is higher than that in Condition \( NoA4 \) (\( NoA8 \)).

Note that the above hypothesis is written in a weak sense as it only compares the relative individual contributions.

### Table 1: Experimental Treatments

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Group Size</th>
<th>First 15 Rounds</th>
<th>Second 15 Rounds</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( N = 4 )</td>
<td>Condition ( A4 ): Linear Public Goods Game with Endogenous Authority and Punishment</td>
<td>Condition ( NoA4 ): Linear Public Goods Game with No Authority</td>
<td>( A-NoA )</td>
</tr>
<tr>
<td>2</td>
<td>( N = 8 )</td>
<td>Condition ( A8 ): Linear Public Goods Game with Endogenous Authority and Punishment</td>
<td>Condition ( NoA8 ): Linear Public Goods Game with No Authority</td>
<td>( NoA-A )</td>
</tr>
<tr>
<td>3</td>
<td>( N = 4 )</td>
<td>Condition ( NoA4 ): Linear Public Goods Game with No Authority</td>
<td>Condition ( A4 ): Linear Public Goods Game with Endogenous Authority and Punishment</td>
<td>( NoA-A )</td>
</tr>
<tr>
<td>4</td>
<td>( N = 8 )</td>
<td>Condition ( NoA8 ): Linear Public Goods Game with No Authority</td>
<td>Condition ( A8 ): Linear Public Goods Game with Endogenous Authority and Punishment</td>
<td>( NoA-A )</td>
</tr>
</tbody>
</table>

### Table 2: Summary of Theoretical Predictions

<table>
<thead>
<tr>
<th>Experimental Conditions</th>
<th>( NoA4 )</th>
<th>( A4 )</th>
<th>( NoA8 )</th>
<th>( A8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Group Size ( N )</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>(2) Individual Contest Spending ( x^* )</td>
<td>–</td>
<td>2.727</td>
<td>–</td>
<td>1.691</td>
</tr>
<tr>
<td>(3) Individual Public Goods Provision ( y^* )</td>
<td>0</td>
<td>47.273</td>
<td>0</td>
<td>48.309</td>
</tr>
<tr>
<td>(4) Total Public Goods Provision ( (N - 1) \cdot y^* )</td>
<td>0</td>
<td>141.819</td>
<td>0</td>
<td>338.163</td>
</tr>
<tr>
<td>(5) Mean Punishment ( \frac{\alpha + \pi}{2} )</td>
<td>–</td>
<td>22.273</td>
<td>–</td>
<td>26.328</td>
</tr>
<tr>
<td>(6) Efficiency Gain ( B ) (%)</td>
<td>–</td>
<td>70.909</td>
<td>–</td>
<td>84.541</td>
</tr>
</tbody>
</table>
public good contribution levels in different conditions rather than stating the absolute levels. In particular, under conditions NoA4 and NoA8, it is predicted that the individual public good contribution is zero. However, it has been shown in the literature that people tend to contribute positive amounts to public goods in the lab even without any possibility of punishment.

Our next two hypotheses are developed around the comparative statics with respect to the group size. Based on the result in Proposition 1 and the numerical results presented in Table 2, we have the following hypotheses related to the effect of group size on the public goods contribution and the efficiency rate (social cost), and on the contest spending.

**Hypothesis 2.** For a given order of conditions, both the public goods contribution and the efficiency rate in Condition A8 are higher than those in Condition A4.

**Hypothesis 3.** For a given order of conditions,

(a) the individual contest spendings in both Conditions A4 and A8 are positive, and

(b) the individual contest spending in Condition A8 is lower than that in Condition A4.

For the authority’s behavior, according to Table 2, it is necessary for at least the minimum punishment $s$ to be handed down to the minimum contributor in the group. Moreover, the minimum punishment in Condition A4 is less severe than that in Condition A8. At the same time, the authority has no ex-post incentive to contribute to the public goods. These are summarized as follows:

**Hypothesis 4.**

(a) The punishment handed down by the authority in Condition A8 is more severe than that in Condition A4.

(b) The authority contributes nothing to the public goods.

### 3.2 Experimental Procedures

The experiments were conducted using z-Tree (Fischbacher, 2007) at the Hong Kong University of Science and Technology (HKUST). For each treatment, four sessions were conducted for games with $N = 8$ and two sessions were conducted for games with $N = 4$. Each session for games with $N = 8$ has two groups of eight subjects; each session for games with $N = 4$ has four groups of four subjects. The subjects were divided into two matching classes. This ensures that we had 4 independent observations for each group size.\(^{16}\) We used a random matching protocol for each matching class.

\(^{16}\)Note that having more than four independent observations increases statistical significance as long as measurement error is small enough (Loken and Gelman, 2017).
Subjects with no prior experience with our experiments were recruited from the undergraduate population in the HKUST. In total, 192 subjects participated in the 12 sessions. Upon arrival at the lab, subjects were instructed to sit at separate computer terminals, and each received a copy of the instructions for the experiment. The instructions were then read aloud. The subjects were asked to answer a series of questions related to the instructions on their own and then the answers were revealed to them. This ensured that the information in the instructions was induced as public knowledge among the participants. In each session, subjects were given two rounds of practice before commencing the 30 official rounds (first 15 and second 15 rounds).

We take the game with the order NoA-A to illustrate the experimental procedure. Full instructions and screen shots are in the Appendix A. In each of the first 15 rounds, each subject was endowed with 50 tokens and was asked to divide the 50 tokens between two investment opportunities: BLUE investment and RED investment. Any division of 50 with integer numbers between 0 and 50 (inclusive) was allowed. The two investment opportunities gave different returns. Each token a subject allocated to the BLUE investment earned him/her a return of 2 points. Each tokens allocated to the RED investment earned every group member a return of 1.5 points.

Each of the second 15 rounds consisted of two stages. At the beginning of Stage 1 (Administrator Fund Stage), subjects were endowed with 50 tokens and asked to decide how many tokens to invest in the Administrator Fund (AF). Any integer number between 0 and 50 (inclusive) was allowed. For each token invested in AF, a subject receives one lottery ticket with a number on it. Once all group members had submitted their investment decision, the computer randomly selected one winning lottery ticket (winning number). The subject with the winning lottery ticket was made the administrator/authority of the group in that round.

After the first stage AF decision, no matter whether a subject became the administrator or not, he/she was asked to divide the remaining tokens (50 - number invested in AF) between two investment opportunities: BLUE investment and RED investment. The returns were exactly the same as in the first 15 rounds. After the investment decisions were made, the administrator was informed of the decisions of group members. The administrator had the power to impose a fine on group members subject to two constraints: (1) The administrator has to fine only one of the group members, and (2) the size of the fine should be no less than 0 and no more than three times the total size of the AF, i.e. (3× the sum of AF spending by a group). At the end of each round, the computer provided a summary of the individual AF spending, BLUE investment, group total RED investment, fine imposed if any, and the final income.

The experimenter randomly selected two rounds to calculate cash payment: one from the first 15 rounds and one from the second 15 rounds. The total payment was the sum of the incomes in the two selected rounds (divided by five) plus a HK$30 show-up fee. The experiments lasted around 100 minutes. Subjects earned on average HK$128.95 (≈ US$16.63).
4 Experimental Findings

Our analysis focuses on comparing the performance of public good provision when there is an endogenous authority with that when there is no authority. In addition, we investigate the impact of having an endogenous authority and punishment in groups of different sizes. We also analyze the behavior of subjects in the contest to become the authority and how the authority contributes to the public goods and hands down punishment. Some dynamic analyses of contest spending, authority, and punishments are reported in Appendix A.

4.1 Public Goods Contribution and Earnings

Figure 2 shows that allowing for an endogenous authority leads to a significant increase in public goods contribution in both conditions with group sizes equal to 4 and 8. For both orders $A-NoA$ (Figure 2(a)) and $NoA-A$ (Figure 2(b)), the average contribution to public goods in the larger group is significantly higher than that in the smaller group. Using the average contributions of public goods at the four independent matching class levels, we conduct four nonparametric Wilcoxon signed-rank tests to compare the average public goods contribution levels between 1) the treatment condition of $A_4$ and of $NoA_4$ in the order $A-NoA$, 2) the treatment condition of $A_4$ and of $NoA_4$ in the order $NoA-A$, 3) the treatment condition of $A_8$ and of $NoA_8$ in the order $A-NoA$, and 4) the treatment condition of $A_8$ and of $NoA_8$ in the order $NoA-A$. The tests reveal that we can reject the null hypothesis that, for a given order, there is no difference in the public contributions between $A_4$ and $NoA_4$ and between $A_8$ and $NoA_8$ in favor of the alternative that the public contribution is bigger in the conditions with authority ($p = 0.0679$ for all four tests).\(^{17}\)

![Figure 2: Average Public Contribution](image)

\(^{17}\) We test our hypotheses mainly using non-parametric tests with matching class level data as independent observations. However, for the robustness checks, we also provide results from regression analyses with robust standard errors clustered at the matching class level.
Average earnings, as shown in Figure 3, are significantly higher in A8 than in NoA8 regardless of the order \((p = 0.0679, \text{ both Wilcoxon signed-rank tests})\). In the conditions with a group size of 4, the average earnings with and without endogenous authority are significantly different for the order A-NoA \((p = 0.0947, \text{ Wilcoxon signed-rank test})\) but insignificantly for the order NoA-A \((p = 0.715, \text{ Wilcoxon signed-rank test})\). The findings are consistent with our theoretical prediction that the benefit of having an endogenous authority is larger for a larger group. For a smaller group, the social cost of competing for authority can be relatively large, even though people on average contribute more to the public goods in the presence of authority.

The results from the regression analysis using the random effects model confirm those from the summary statistics, as shown in Table 3. According to column (2) of Table 3, subjects in the conditions with a group size of 4 and with authority (A4) gave on average 5 more tokens than those in the conditions with the same group size with no authority (NoA4). In Condition A8, the average contribution to the public goods is even higher (around 9.5 tokens more) than that in the baseline condition. These findings are robust to the use of all observations from the two stages of the experiment and to controlling for the impact of authority status, period, and order of conditions.

The endogenous authority has a similar but stronger effect on the contribution rate, defined as the
proportion of the endowment remaining after the contest spending that is contributed to the public goods.\textsuperscript{18} The contribution rates in the conditions with endogenous authority are 19\% and 10\% more for the larger group and the smaller group, respectively, than those in the conditions without authority. These findings are robust across different specifications, as shown in Table 3.

Table 3: Determinants of Contributions and Earnings (Random Effects Model)

<table>
<thead>
<tr>
<th>Model</th>
<th>Contribution level</th>
<th>Contribution rate</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First (1)</td>
<td>First (3)</td>
<td>First (5)</td>
</tr>
<tr>
<td>Sample</td>
<td>Full (2)</td>
<td>Full (4)</td>
<td>Full (6)</td>
</tr>
<tr>
<td>Condition A8</td>
<td>9.531**</td>
<td>0.191**</td>
<td>95.11</td>
</tr>
<tr>
<td></td>
<td>(4.591)</td>
<td>(0.0918)</td>
<td>(66.03)</td>
</tr>
<tr>
<td>Condition A4</td>
<td>5.099*</td>
<td>0.102*</td>
<td>40.98</td>
</tr>
<tr>
<td></td>
<td>(2.731)</td>
<td>(0.0546)</td>
<td>(27.84)</td>
</tr>
<tr>
<td>Condition NoA8</td>
<td>2.236</td>
<td>0.0447</td>
<td>40.98</td>
</tr>
<tr>
<td></td>
<td>(1.527)</td>
<td>(0.0305)</td>
<td>(9.256)</td>
</tr>
<tr>
<td>Authority</td>
<td>-18.64***</td>
<td>-0.373***</td>
<td>44.99***</td>
</tr>
<tr>
<td></td>
<td>(1.627)</td>
<td>(0.0325)</td>
<td>(4.958)</td>
</tr>
<tr>
<td>Period</td>
<td>-0.612***</td>
<td>-0.0122***</td>
<td>-4.942***</td>
</tr>
<tr>
<td></td>
<td>(0.0993)</td>
<td>(0.00199)</td>
<td>(1.030)</td>
</tr>
<tr>
<td>Period×A</td>
<td>0.746***</td>
<td>0.0149***</td>
<td>5.816***</td>
</tr>
<tr>
<td></td>
<td>(0.157)</td>
<td>(0.00314)</td>
<td>(1.386)</td>
</tr>
<tr>
<td>order</td>
<td>1.746</td>
<td>0.00849</td>
<td>31.83</td>
</tr>
<tr>
<td></td>
<td>(1.800)</td>
<td>(0.0367)</td>
<td>(30.96)</td>
</tr>
<tr>
<td>Constant</td>
<td>18.59***</td>
<td>0.372***</td>
<td>194.3***</td>
</tr>
<tr>
<td></td>
<td>(1.687)</td>
<td>(0.0337)</td>
<td>(10.16)</td>
</tr>
<tr>
<td>Observations</td>
<td>2,880</td>
<td>2,880</td>
<td>2,880</td>
</tr>
<tr>
<td>Number of id</td>
<td>192</td>
<td>192</td>
<td>192</td>
</tr>
</tbody>
</table>

Notes: 1. Condition A8 refers to the treatment with a larger group size (8) and with authority; Condition A4 represents the treatment with a smaller group size (4) and with authority; Condition NoA8 is the treatment with a larger group size (8) but without authority.
2. Authority is a dummy that is equal to 1 if a subject was selected as the authority; 0 otherwise. Period×A is the interaction term between period and treatment with authority.
3. “First” represents the data from the first 15 rounds only whereas “Full” represents data from all rounds.
4. Robust standard errors are in parentheses, clustered at the matching class level.
5. Significance level: *** p<0.01, ** p<0.05, * p<0.1.

Confirming our Hypothesis 1, we summarize the finding on public goods contribution as the following:

Result 1. Public goods contributions in the conditions with an endogenous authority are significantly higher than those in the conditions without authority.

\textsuperscript{18}There is no single round in which the endogenous authority did not emerge in our data.
<table>
<thead>
<tr>
<th></th>
<th>NoA4</th>
<th>A4</th>
<th>NoA8</th>
<th>A8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Group Size $N$</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>(2) Individual Contest Spending</td>
<td>–</td>
<td>6.76</td>
<td>–</td>
<td>4.63</td>
</tr>
<tr>
<td>(3) Individual Public Goods Provision</td>
<td>12.64</td>
<td>20.43</td>
<td>16.64</td>
<td>23.83</td>
</tr>
<tr>
<td>(4) Total Public Goods Provision</td>
<td>50.56</td>
<td>81.72</td>
<td>131.20</td>
<td>190.64</td>
</tr>
<tr>
<td>(5) Punishment</td>
<td>–</td>
<td>62.55</td>
<td>–</td>
<td>96.76</td>
</tr>
<tr>
<td>(6) Efficiency Gain (%)</td>
<td>–</td>
<td>40.86</td>
<td>–</td>
<td>47.65</td>
</tr>
<tr>
<td>(7) Social Cost (%)</td>
<td>–</td>
<td>32.20</td>
<td>–</td>
<td>18.95</td>
</tr>
<tr>
<td>(8) Earnings</td>
<td>150.58</td>
<td>153.54</td>
<td>266.46</td>
<td>317.37</td>
</tr>
</tbody>
</table>

Notes: 1. Empirical Efficiency Gain (%) = $100 \times \frac{Total\ Public\ Goods\ Contribution}{N\omega}$
2. Empirical Social Cost (%) = $100 \times \frac{Total\ Contest\ Spending + \omega - \text{Authority's Public Goods Contribution}}{N\omega}$

Our second hypothesis is about the effect of group size on the performance of an endogenous authority. As shown in Table 4, in the conditions with authority, the average public goods contribution in the larger group (23.83) is higher than that in the smaller group (20.43), but the difference is significant only for the order $A-NoA$ ($p = 0.0679$ for the order $A-NoA$ and $p = 0.4652$ for the order $NoA-A$, Wilcoxon signed-rank tests).

Figure 4 shows that the higher efficiency gain in the larger group exists regardless of the order. Similarly, according to Table 4, the efficiency gain in the larger group is 47.65% and is higher than that (40.86%) in the smaller group. Overall, the difference in the empirical efficiency gain between two groups of different sizes is only moderate, and is not as substantial as theoretically predicted.

Wilcoxon signed-rank tests ($p = 0.0679$ for both orders) reveal that the average social cost in the larger group $A8$ (18.95%) is significantly lower than that in the smaller group $A4$ (32.20%). The empirical social cost of endogenizing the authority is consistent with the theoretical prediction with only a slight downward bias, which might come from the fact that authorities also contribute to the public goods in the experiments we conducted. These results reject our Hypothesis 2 and are summarized as the following:

**Result 2.** *Comparing the conditions with an endogenous authority in the smaller group and the larger group, the public goods contribution and the efficiency gains are not significantly different while the social cost is significantly lower in the larger group.*

Note that the typical pattern of cooperative decay is evident in the $NoA$ treatments but not in the $A$ treatments (see Figures 2, 3, and 4). Our regression result reported in Table 3 confirms this observation. The interaction term between treatment and period has significant and positive coefficients while the period term has significant and negative coefficients, showing that the negative effect of period is driven by the $NoA$ treatments.
4.2 Contest Spending, Punishment, and Authority

One of the important premises of our analysis is that the authority behaves according to the following two conditions: 1) the authority chooses the lowest contributor among non-authority members to punish and 2) when punishing the lowest contributor, the authority uses enough punishment. In this section, we first scrutinize if these two conditions are satisfied. We further discuss the contest spending and authority’s contribution levels.

![Graph showing frequency of the lowest contributor being punished](image1)

Figure 5: Frequency of Lowest Non-authority Contributor Being Punished

Figure 5 presents the frequencies of the lowest contributor among non-authority members being chosen to be punished.\(^\text{19}\) Overall, the frequency is quite high and stable regardless of the orders and group sizes. The authorities chose the lowest non-authority contributor to punish 201 times out of 240 (83.8\%) in the conditions with group size of 4 and 189 times out of 240 (78.8\%) in the conditions with group size of 8. Figure 6 shows that majority of authorities fully exercise their punishment power where the punishment ratio is defined as the ratio between the full amount of punishment available for authorities and the actual amount of punishment exercised. The fractions of authorities who exercise more than 90\% of the full amount of punishment are about 55\% in the conditions with group size of 4 and over 75\% in the conditions with group size of 8. Taking the two results together, we conclude that the authorities’ behavior in our data is qualitatively consistent with the prediction from the equilibrium we concentrated on in the theory section.

In the competition for authority, subjects spend significantly less per person in Condition A8 than that in A4 for the order A-NoA \((p = 0.0679, \text{ Wilcoxon signed-rank test})\) but the significance disappears for the order NoA-A \((p = 0.2733, \text{ Wilcoxon signed-rank test})\). Row 5 of Table 4 shows that the average punishment is significantly more severe in Condition A8 than in A4 for the order A-NoA \((p = 0.0679, \text{ Wilcoxon signed-rank test})\) but only insignificantly more for the order NoA-A \((p = 0.4652, \text{ Wilcoxon signed-rank test})\). This is a direct consequence of the fact that the authority has greater punishment power from the bigger total contest spending in the larger group.

\(^{19}\)When there are two or more people tied for the lowest contributor, which happened very rarely in our data, all are indexed and counted as the lowest contributors.
On average, each individual in conditions $A4$ and $A8$ spent 6.76 tokens and 4.63 tokens to win the contest, which are significantly higher than the corresponding theoretical predictions ($p = 0.021$, Wilcoxon rank-sum test).

Figures 7 and 8 present the time trend data for contest spending and punishment. The patterns are consistent with the nonparametric test results. Confirming our Hypotheses 3 and 4, we summarize the results related to contest spending and punishment as follows.

**Result 3.** *Average contest spending in the condition with the larger group is significantly lower than that in the condition with the smaller group, whereas the average punishment in the condition with the larger group is significantly more severe than that in the condition with the smaller group.*
predicts—the authority has no incentive to contribute to the public goods because he/she will not be punished anyway.

Social rent exists for the authority as the authority earns significantly more than others. According to Table 5, the average earnings for authority and other people are 223 and 206 in the condition with group of size 4, and are 313 and 299 in the condition with group of size 8. The regression results in columns (5) and (6) of Table 3 show that authorities earn on average 44.99 and 40.19 HKD more than others for the between-subjects comparison and the comparison using the whole sample.

The following summarizes our findings on the behavior of authorities.

**Result 4.** **Authorities contribute positively to the public goods even without the possibility of being punished. They earn significantly more than non-authority subjects.**

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20The payoff of authority includes its own endowment after contest spending plus the benefit from the public goods, but does not include the points unused in the punishment pool.
Table 5: Comparisons between Authority and Non-Authority

<table>
<thead>
<tr>
<th>Group</th>
<th>Member</th>
<th>Statistic</th>
<th>Contribution Level</th>
<th>Contest Spending</th>
<th>Contribution Rate</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>N0A4</td>
<td>Non-authority</td>
<td>Mean (Median)</td>
<td>13.4 (10)</td>
<td>-</td>
<td>.27 (.2)</td>
<td>187 (175)</td>
</tr>
<tr>
<td>A4</td>
<td>Authority</td>
<td>Mean (Median)</td>
<td>24.7 (23)</td>
<td>4.34 (2)</td>
<td>.53 (.5)</td>
<td>206 (183)</td>
</tr>
<tr>
<td>N0A8</td>
<td>Non-authority</td>
<td>Mean (Median)</td>
<td>16.64 (10)</td>
<td>-</td>
<td>.33 (.2)</td>
<td>257 (258)</td>
</tr>
<tr>
<td>A8</td>
<td>Authority</td>
<td>Mean (Median)</td>
<td>26.78 (25)</td>
<td>3.83 (1)</td>
<td>.56 (.5)</td>
<td>299 (309.75)</td>
</tr>
</tbody>
</table>

5 Discussion and Concluding Remarks

This paper investigates theoretically and experimentally the social benefits and costs of endogenizing a punishment-enforcing authority in the linear public goods framework. Prior to the standard public goods game an authority is chosen endogenously among members of a society via the imperfectly discriminating contest. Once chosen the authority has the complete freedom to enforce punishment. We show that the endogenous authority can approximate the fully efficient outcome for a large society, although endogenizing the authority involves a social cost.

Our experimental results are qualitatively consistent with most of the theoretical findings. We find that the endogenous authority can increase the contributions to public goods significantly. The contribution to the public goods and the efficiency gains are higher in the treatment with a larger group than those in the treatment with a smaller group. Moreover, the social cost is significantly lower in the treatment with a larger group. A larger group size reduces the contest spending, but leads to more severe punishment. Authorities still contribute positively to the public goods even though they could never be punished.

As mentioned earlier, the real costs the society has to bear in endogenously choosing the authority would be higher than what we have considered in our model. For example, people may over-spent in the contest due to the positive intrinsic value of the authority (Fehr et al., 2013); the authority abuses his/her power as in Muthukrishna et al. (2017) and results in social welfare loss; members of the society hesitate to comply with the authority when legitimacy of the authority is not fully established (Dickson et al., 2017). Investigating some of these possibilities in theory and in experiments will be an interesting extension of our paper.

There are a few limitations of our study. First, one peculiar restriction in our punishment mechanism is that the authority needs to choose only one member to punish. In theory, this restriction is non-binding as only one member will be punished in equilibrium even when punishing more than one member is allowed. However, it turns out that this condition becomes quite crucial in the experiment.
We had a few pilot experiments in which the authority has freedom to choose as many people to punish as he/she wants under the second constraint. The data show that choosing only one member to punish does not come out endogenously, and sometimes the resulting punishment amount applied to each individual being punished becomes not sufficiently high to deter the deviation to free-riding. Second, we acknowledge that there is a fair amount of casual ambiguity in our experimental design. We know that the proposed mechanism as a whole increases contributions, but exactly which features of the mechanism plays a key role is not crystal clear.

One interesting product of our mechanism is that, while our authority mechanism can sustain cooperation, it also leads to larger inequality across group members. The concern of inequality becomes weaker as the group size increases. These observations give us an important implication for whether a society of different sizes would endogenously choose to organize public goods under such a regime or rather prefer unregulated peer punishment (e.g., Fehr and Gachter (2000)) or more democratic pool punishment institutions (e.g., Ozono et al. (2016)). An experimental study that investigates the endogenous choice of a punishment regime to implement when a few alternative mechanisms are available will be an interesting future research. It would be also interesting and important to design experiments that can directly compare an endogenous authority with exogenous institutions. Another direction is to consider a democratic way of selecting the authority who might contribute to the public goods more than the authority in our setting. One can also impose election procedures without costly contests as that in Hamman et al. (2011), where the central issue is how to induce the authority to be accountable and committed to law enforcement. We leave these issues for future research.
References


Fehr, Ernst and Tony Williams (2018), “Social norms, endogenous sorting and the culture of cooperation.”


A Appendix - Dynamic Analysis

In this appendix, we document the dynamics of contest spending, punishment, and contributions. Precisely, we provide aggregate level overviews on 1) who wins the contest and becomes authority, 2) how individual winners vary in their behavior as the authority, 3) whether past winners or losers increase or decrease their contest spending, and 4) whether individuals who were punished in the past increase or decrease their contest spending.

Who wins the contest and becomes authority? Figure 10 presents the distributions of number of winning in the pre-LPG contests. As shown in the figure, more people in the treatments with a larger group size (around 37%) have never been the authority than in the treatments with a smaller group size (around 10%). One remarkable observation is that there are a few individuals who repeatedly became the authority in both treatments. Thus, we would like to see if this observation is driven by their aggressiveness in the pre-LPG contests.

How individual winners vary in their behavior as the authority? Figure 11 reports the distributions of authorities’ contest spending. It is immediately clear that these subjects behaved very aggressively in the pre-LPG contests relative to the equilibrium contest spending (2.727 for Treatment A4 and 1.691 for Treatment A8).
Past contest winners and losers behaved differently in terms of their contest spending. As shown in Figure 12, past contest winners spent significantly more in the later contests than those who lost the contest. 48% percent of losers spent zero in next round of contest and only 12% of winners spent zero. This result implies that learning across different rounds took place in the experiments.

Figure 12: Spending in Contest for Past Winners and Losers Authority

Do past winners or losers increase or decrease their contest spending? Given that learning is an important determinant of the contest spending, we also expect to observe a positive correlation between individual’s experience of being punished and their contest spending when they are not the authority. Confirming this conjecture, Figure 13 shows that the contribution rate increased significantly right after individuals were punished relative to their contribution rate before they got punished.

Figure 13: Contribution Rate and Punishment

Do individuals who were punished in the past increase or decrease their contest spending?
Appendix - Instruction for *Treatment 4*

**INSTRUCTION**

Welcome to the experiment. This experiment studies decision making between eight individuals. In the following two hours or less, you will participate in 30 rounds (first 15 rounds and second 15 rounds) of decision making. Please read the instructions below carefully; the cash payment you will receive at the end of the experiment depends on how well you make your decisions according to these instructions. If you have a question at any point, please raise your hand and wait for one of us to come over. We will then privately answer your question. We ask that you turn off your mobile phone and any other electronic devices. Communication of any kind with other participants is not allowed.

**Your Group**

In each and every round, you will be *randomly* matched with seven other participants to form a group of eight. You will not be told the identity of the participants you are matched with, nor will those participants be told your identity—even after the end of the experiment. The eight members in a group make decisions that will affect their rewards in the round. Participants will be randomly rematched after each round to form new groups.

**First 15 Rounds: BLUE and RED Investment Decision**

In each round, you will be endowed with 50 tokens and asked to decide how to divide 50 tokens into two investment opportunities; BLUE investment and RED investment. You input your investment decision by clicking on a line with a green ball on it (see Figure 14). The left end of the line represents 0 tokens in BLUE investment and 50 tokens in RED investment; the right end of the line represents 50 tokens in BLUE investment and 0 tokens in RED investment. You can choose any integer point in between. When you click on the line, the green ball will move to the point you click on, and the corresponding numbers of BLUE investment and RED investment will be shown inside BLUE BOX and RED BOX above the line. You adjust
your click until you arrive at your desired numbers, after which you click the submit button. Your decision in the round is then completed.

The two investment opportunities give different returns. The following shows returns from each investment.

- **BLUE investment**: Each token you invest in the BLUE investment will earn you a return of 2 points.
- **RED investment**: What you earn from the RED investment will depend on the total number of tokens that you and the other seven members of your group invest in the RED investment. Each token your group invest in the RED investment will earn you a return of 1.5 points.

More precisely,

\[
\text{Your income} = 2 \times (\text{Your BLUE investment}) + 1.5 \times (\text{Your group total RED investment}).
\]

Here are some examples:

1. Suppose that you invest 20 tokens in the BLUE investment and 30 tokens in the RED investment. Suppose that your group members (including yourself) together invest 90 tokens in the RED investment. Your income in this case is:

   \[
   \left[20 \times 2\right] + \left[90 \times 1.5\right] = 175.
   \]

2. Suppose that you invest 50 tokens in the BLUE investment and 0 tokens in the RED investment. Suppose that your group members (including yourself) together invest 90 tokens in the RED investment. Your income in this case is:

   \[
   \left[50 \times 2\right] + \left[90 \times 1.5\right] = 235.
   \]

**Second 15 Rounds: Two-stage Investment Decision**

**Stage 1: Administrator Fund (AF)**

At the beginning of each round, you will be endowed with 50 tokens and will be asked to decide how many tokens to spend on Administrator Fund (AF). Any integer amount between 0 and 50 (inclusive) is possible. In return to 1 token spending on AF, you will receive 1 lottery ticket with a ticket number on it. After all group members finish spending on AF, the computer randomly selects one winning lottery ticket (winning number). Each lottery ticket has equal chance to be selected.

If you have the winning lottery ticket (the winning number), then you become an administrator of your group in the round. After the 2nd stage, the administrator will monitor your group members’ RED investment and will have authority to impose a payoff deduction to one of the group members. More details will be provided later.
Stage 2: BLUE and RED Investment Decision

After the 1st stage Administrator Fund decision, no matter whether you become the administrator or not, you will be asked to decide how to divide remaining tokens (50 - Your Spending on AF) into two investment opportunities: BLUE investment and RED investment. The procedure of investment and returns from each investment are exactly the same as the first 15 rounds.

(a) Stage 1: Administration Fund

(b) Stage 2: Investment Decision

ADMINISTRATOR

The following screen shows how the administrator examines your RED investment.

(a) Administrator Screen 1

(b) Administrator Screen 2

The administrator may impose a deduction from your income subject to the following constraints:

1. The administrator needs to select only one of the group members to impose a payoff deduction.

2. The size of payoff deduction imposed to a group member should be no less than 0 and no more than three times the total size of the Administrator Fund, i.e. \((3 \times \text{the sum of AF spending in a group})\).
Your final income = 2 × (Your BLUE investment) + 1.5 × (Your group total RED investment) − (Payoff Deduction).

Here are some examples that show how to calculate your final income in each round:

1. Suppose that you spend 10 tokens on the Administrator Fund and get 10 lottery tickets with ticket number 11-20. It turns out that the winning lottery ticket number is 29. Then you are not an administrator. Next, you invest 10 tokens in the BLUE investment and 30 tokens in the RED investment. Your group members (including yourself) together invest 80 tokens in the RED investment. Suppose the size of AF in your group is 48 so that the power of administrator becomes 144. In the end, the administrator in your group imposes a payoff deduction of 30 to you. Your income in this round is:

\[
\frac{10 \times 2}{\text{BLUE Investment}} + \frac{80 \times 1.5}{\text{Group RED Investment}} - \frac{30}{\text{Payoff Deduction}} = 110.
\]

2. Suppose that you spend 10 tokens on the Administrator Fund and get 10 lottery tickets with ticket number 11-20. It turns out that the winning lottery ticket number is 29. Then you are not an administrator. Next, you invest 10 tokens in the BLUE investment and 30 tokens in the RED investment. Your group members (including yourself) together invest 80 tokens in the RED investment. Suppose the size of AF in your group is 100 so that the power of administrator becomes 300. In the end, the administrator in your group imposes a payoff deduction of 200 to you. Your income in this round is:

\[
\frac{10 \times 2}{\text{BLUE Investment}} + \frac{80 \times 1.5}{\text{Group RED Investment}} - \frac{200}{\text{Payoff Deduction}} = -60 \text{ (rounded up)} \Rightarrow 0
\]

Since your income is less than 0 in this case, you will just get 0 income.

3. Suppose that you spend 20 tokens in the Administrator Fund and get 20 lottery tickets with ticket number 26-45. It turns out that the winning lottery ticket number is 42. Then you become an administrator. Next, you invest 30 tokens in the BLUE investment and 0 tokens in the RED investment. Your group members (including yourself) together invest 50 tokens in the RED investment. Suppose the size of AF in your group is 60 so that the power of administrator becomes 180. In the end, the administrator (yourself) in your group imposes a payoff deduction of 0 to you. Your income in this round is:

\[
\frac{30 \times 2}{\text{BLUE Investment}} + \frac{50 \times 1.5}{\text{Group RED Investment}} - \frac{0}{\text{Payoff Deduction}} = 135.
\]

**Note.** The amount taken by the administrator will disappear and no one, including the administrator, can get it.

**Information Feedback**
At the end of each round, the computer will provide a summary for the round: your AF spending, your BLUE investment, your group total RED investment, payoff deduction imposed to you, and your final income in the round.

**Your Cash Payment**

The experimenter randomly selects 2 rounds to calculate your cash payment, one from the first 15 rounds and one from the last 15 rounds. Each round has equal chance to be selected (So it is in your best interest to take each round seriously.) Your total payment will be the sum of your incomes in the two selected rounds divided by five plus a HK$30 show-up fee. Precisely,

\[
\text{Your final cash payment} = \text{HK$} \left[ \frac{\text{Sum of your incomes in two randomly selected rounds}}{5} \right] + 30
\]

**Practice Rounds**

To ensure your understanding of the instructions, we will provide you with a quiz and practice rounds. We will go through the quiz after you answer it on your own. You will then participate in 2 practice rounds, one for the first 15 round and one for the second 15 round. The practice rounds are part of the instructions which are not relevant to your cash payment; its objective is to get you familiar with the computer interface and the flow of the decisions in each round.

Once the practice rounds are over, the computer will tell you “The first 15 rounds begin now!” Once the first 15 rounds are over, the computer will tell you ‘The second 15 rounds begin now!’ In each round, you will be randomly matched with seven other participants in this room.

**Administration**

Your decisions as well as your monetary payment will be kept confidential. Remember that you have to make your decisions entirely on your own; please do not discuss your decisions with any other participants. Upon finishing the experiment, you will receive your cash payment. You will be asked to sign your name to acknowledge your receipt of the payment (which will not be used for tax purposes). You are then free to leave.

If you have any question, please raise your hand now. We will answer your question individually. If there is no question, we will proceed to the quiz.

**Quiz**

1. True or False: I will be matched with the same seven other players in all 30 rounds.

2. In one of the first 15 rounds, suppose that you invest 20 tokens in BLUE investment and 30 tokens in RED investment. Your group total RED investment is 100 tokens. Calculate your income in this round.---------
3. In one of the second 15 rounds, suppose that you invest 20 tokens in BLUE investment and 10 tokens in RED investment. Your group total RED investment is 100 tokens. It turns out that the administrator imposes a payoff deduction of 40 tokens for you. Calculate your income in this round. 

4. True or False: You spent 20 tokens for the AF. Your group members (including yourself) together spent 100 tokens for the AF. Then you have a 20 in 100 chance of being an administrator.

5. True or False: Your group members (including yourself) together spent 100 tokens for the AF. The administrator in your group can impose a payoff deduction for one of your group members no more than 300 tokens.