

# On Identifying Higher-Order Rationality\*

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## Abstract

We study how to identify the exact levels of higher-order rationality of players in games, the same question considered in Kneeland [13, *Econometrica*, 83, (2015), 2,065–2,079]. We first present experimental evidence that the exclusion restriction (ER) assumption in Kneeland [13], which is crucial for her identification, is violated in a number of games. We then provide an alternative identification approach that requires an assumption that is not only applicable to any games but also weaker than the ER assumption. Finally, we propose a simple *chain game* to implement our identification approach and report experimental findings from the laboratory.

*Keywords: Rationality, higher-order rationality, revealed rationality, chain game*

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# 1 Introduction

A player is 1st-order rational if she plays a best action in response to her belief regarding payoffs and her opponent's actions. Furthermore, inductively, for any positive integer  $k$ , a player is  $(k + 1)$ -order rational if she believes that her opponents are  $k$ -order rational, and she plays a best action in response to her belief. Common knowledge of rationality means that players are rational, they know that their opponents are rational, and they know that their opponents know that their opponents are rational ad infinitum, i.e., the players are  $\infty$ -order rational.

Nearly all game-theoretical analyses are based on the assumption of common knowledge of rationality (i.e., infinite orders of rationality). However, empirical and experimental results show that many players have *finite* orders of rationality (e.g., Burchardi and Penczynski [4], Costa-Gomes, Crawford, and Iriberri [6], and Costa-Gomes and Weizsäcker [7]). More importantly, players with different levels of rationality choose to play different actions in many games (e.g., the e-mail game; see Rubinstein [15], Dekel, Fudenberg and Morris [8]). Hence, to understand players' strategic behaviors, it is of fundamental importance to identify players' orders of rationality.

One direct approach to identifying players' orders of rationality is to elicit subjects' first-order beliefs and associate them with the choice data (refer to, e.g., Costa-Gomes and Weizsäcker [7] and Healy [12]), which reveals whether subjects are 1st-order rational. However, extending this approach to identifying higher-order rationality would necessarily involve eliciting complicated higher-order beliefs (e.g., beliefs about payoffs, beliefs about their opponents' beliefs about payoffs), which is clearly not tractable. To circumvent this technical problem, a second approach (e.g., Burchardi and Penczynski [4], Costa-Gomes, Crawford, and Iriberri [6]) imposes structural assumptions on players' first-order beliefs (on payoff), which would pin down the behaviors of  $k$ -order rational players for all  $k = 0, 1, 2, \dots$ . As a result, players' higher-order rationality can be identified by looking at their choice data. However, the success of this approach hinges critically on *ad hoc* structural assumptions about players' first-order beliefs.

Recently, Kneeland [13] proposes a new method to identify the *exact* orders of players' rationality without imposing structural assumptions on players' beliefs. Kneeland proposes the *ring game*, which involves different positions (labeled 1, 2, 3,...) for players. A  $k$ -order rational player has a unique rationalizable action at position  $k'$  in the ring game if and only if  $k' \leq k$ . Kneeland [13] aims to identify the orders of players' rationality by

changing payoffs and assigning players to different positions in the ring game.

Note that higher-order rationality involves *an identification problem*: a researcher who observes a player choosing an action that survives  $k$  rounds of iterative deletion of strictly dominated strategies *cannot* tell the exact order of higher-order rationality because such a behavior may originate from a player who performs only  $k - 1$  or fewer rounds of iterative deletion of strictly dominated strategies. To eliminate the problem, Kneeland [13] imposes an exclusion restriction assumption (hereafter, the ER assumption), meaning that a  $k$ -order rational player does not respond to any payoff change that has no impact on the  $k$  rounds of iterative deletion of strictly dominated strategies. Using only this ER assumption, Kneeland identifies players' higher-order rationality.

In this paper, we first study the validity of the ER assumption in Kneeland [13]. We design experiments to test the ER assumption in games, including the ring game in Kneeland [13]. Our experimental findings reveal that a significant proportion of subjects—77.5% in Kneeland's ring game, 67.5% in the rock-paper-scissor game, 45% in the matching penny game, and 27.5% in the pure-coordination game—violate the ER assumption.

Following Brandenburger, Danieli and Friedenberg [3], we then consider the “revealed-rationality” approach to identifying higher-order rationality. How a  $k$ -order rational agent chooses a final action in a game involves two decision-making steps – the *deletion step* and the *selection step*. The standard *k-order rationality theory* fully describes the deletion step, i.e., a  $k$ -order rational agent performs  $k$  rounds of iterative deletion of strictly dominated strategies. When more than one action survive  $k$  rounds of iterative deletion, the agent should enter the selection step to finalize her choice of action, but the existing theory is completely silent regarding this selection step. Our revealed-rationality approach argues that one could use a player's final action to infer the order “ $k$ ” in the deletion step, only if the selection step is subsumed by the deletion step. This approach does not suffer any loss of generality because a model is suitable if its prediction matches the behaviors of the agents, which echoes the classical “as-if” view of Friedman [10].

We can interpret the revealed-rationality approach in an alternative way. If we observe a player selecting an action that does not survive  $k$  rounds of iterative deletion of strictly dominated strategies, then we immediately conclude that the player is not  $k$ -order rational, i.e., he is at most  $(k - 1)$ -order rational. Thus, by observing a player's behavior in games, we identify an upper bound on the player's level of higher-order rationality. The identification problem discussed above is that a non-trivial lower bound cannot be identified from the observed behaviors. To identify a player's exact level of

higher-order rationality, the revealed-rationality approach simply assumes that the upper bound is equal to the exact level, which we call the “upper-bound” assumption. Although this assumption seems strong, we show in Section 5.1 that the ER assumption implies the upper-bound assumption in the ring game. Nevertheless, the upper-bound assumption does not imply the ER assumption in general. When the ER assumption is not satisfied, Kneeland [13] is completely silent regarding the level of rationality, while the upper-bound assumption (i.e., the revealed-rationality approach) still provides some useful information. In this regard, the upper-bound assumption is weaker than the ER assumption and the revealed-rationality approach can be interpreted as a generalization of Kneeland’s approach in cases where the ER assumption is not satisfied.

Finally, we follow Kneeland’s approach to designing games to identify higher-order rationality. Specifically, we propose the *chain game* in which players’ actions have multiple dimensions, with each dimension being labeled 1st, 2nd, 3rd, and so on. We design the payoffs of the game such that the rationalizable actions of a  $k$ -order rational player have a unique rationalizable element in the  $k'$ -th dimension if and only if  $k' \leq k$ . By manipulating the game payoffs, subjects’ behaviors in the games reveal their orders of (revealed-)rationality. Our experimental results show that 96% of participants are 1-order rational, 63% are 2-order rational, 21% are 3-order rational, and 10% are 4-order rational, which is (approximately) first-order stochastically dominated by the distribution obtained by Kneeland [13], possibly due to learning effects in the ring game.

Our identification approach has three advantages over Kneeland’s approach. First, the upper-bound assumption imposed in our approach is weaker than the ER assumption required by Kneeland [13]. Second, our chain game is more efficient than the ring game in terms of the number of players required to identify higher-order rationality. Third, our game does not suffer from potential learning issues (see the detailed discussion in Section 6).

The remainder of the paper is organized as follows. We first review the identification problem in Section 2, and discuss the identification approach in Kneeland [13] in Section 3. In Section 4, we study the validity of the ER assumption. We then discuss the revealed-rationality approach in Section 5. Finally, we propose the *chain game* to achieve identification in Section 6. Section 7 concludes. Our experimental design and results are presented in the Appendix.

## 2 The Identification Problem

Following Bernheim [2] and Pearce [14], we define  $k$ -order rationality as follows. A game is a tuple  $G = \langle I, A \equiv \times_{i \in I} A_i, (g_i : A \rightarrow \mathbb{R})_{i \in I} \rangle$ , where  $I$  is a finite set of players;  $A_i$  is a finite set of actions for player  $i$ ; and  $g_i$  is the payoff function for player  $i$ . Define  $R_i^0(G) \equiv A_i$ . For any positive integer  $k$ , define  $R_i^k(G)$  inductively as follows.

$$R_i^k(G) := \left\{ a_i \in R_i^{k-1}(G) : \begin{array}{l} \exists \rho \in \Delta \left( \times_{j \neq i} R_j^{k-1}(G) \right) \text{ such that} \\ \sum_{a_{-i} \in \times_{j \neq i} R_j^{k-1}(G)} u_i(a_i, a_{-i}) \times \rho(a_{-i}) \\ \geq \sum_{a_{-i} \in \times_{j \neq i} R_j^{k-1}(G)} u_i(a'_i, a_{-i}) \times \rho(a_{-i}), \forall a'_i \in A_i \end{array} \right\}.$$

That is,  $R_i^k(G)$  is the set of actions that survives  $k$  rounds of iterative deletion of strictly dominated strategies.<sup>1</sup> Define  $R_i^\infty(G) \equiv \cap_{k=0}^\infty R_i^k(G)$ . Note that  $R_i^k(G)$  is monotonically decreasing in  $k$ , i.e.,

$$R_i^0(G) \supset R_i^1(G) \supset R_i^2(G) \supset \dots \supset R_i^\infty(G).$$

“ $k$ -order rationality” is defined as follows.

A player is  $k$ -order rational in game  $G$  (★)

if and only if she would always play actions in  $R_i^k(G)$ .

Suppose that we observe the behavior of a player in a game  $G$ . Based on this observation, can we draw a conclusion such as “the player is  $k$ -order rational” for some  $k$ ? The answer is *no*. To see this, consider the case in which we observe player  $i$  playing  $a \in R_i^{k+1}(G) \subset R_i^k(G)$ . Such an observation cannot help us identify the exact order of rationality for this player: she may be  $(k+1)$ -order rational because  $a \in R_i^{k+1}(G)$ ; alternatively, she may be  $k$ -order but not  $(k+1)$ -order rational, although she chooses  $a \in R_i^{k+1}(G) \subset R_i^k(G)$ . This is the identification problem.

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<sup>1</sup>More precisely,  $R_i^k(G)$  is the set of actions that survives  $k$  rounds of iterative deletion of never-best replies, and a standard min-max argument shows that a strategy is strictly dominated if and only if it is never best.

For instance, consider the following bimatrix game  $G^1$  studied in Kneeland [13].

$$G^1$$

	player 1		player 2	
	player 2's choice		player 1's choice	
player 1's choice		c	d	
	a	15	0	
	b	5	10	
		player 1's choice		
		c	a	b
		10	5	
		d	5	0

Suppose that we observe player 1 playing  $a \in R_1^2(G^1) \subset R_1^1(G^1) = \{a, b\}$ . Although it is plausible that this player is 2-order rational, we can never be certain because such a behavior may originate from a 1-order but not a 2-order rational player.

### 3 Kneeland's Identification

Following Kneeland [13, Footnote 13 on Page 2,069], a player's 0-order payoff is her own payoff; a player's 1-order payoff is her opponent's payoff; a player's 2-order payoff is her opponent's opponent's payoff, and so on. For  $k = 0, 1, 2, \dots$ , let  $h_k^i(G)$  denote the  $k$ -order payoff of player  $i$  in  $G$ . Clearly,  $R_i^1(G)$  is completely determined by  $h_0^i(G)$ ;  $R_i^2(G)$  is completely determined by  $[h_0^i, h_1^i(G)]$ ; and  $R_i^k(G)$  is completely determined by  $[h_0^i(G), \dots, h_{k-1}^i(G)]$ . Based on this observation, Kneeland [13] proposes the following assumption.

**Assumption 1 (Kneeland's ER assumption)** *For any  $k = 1, 2, \dots$ , a  $k$ -order rational player plays the same action in any two games  $G$  and  $G'$  if  $[h_0^i(G), \dots, h_{k-1}^i(G)] = [h_0^i(G'), \dots, h_{k-1}^i(G')]$ .*

The ER assumption is then used by Kneeland [13] to identify a player's order of rationality. Specifically, suppose that we can further observe player 1's action in  $G^2$  below

in addition to her action in  $G^1$  above.

$G^2$

player 1			player 2		
player 2's choice			player 1's choice		
player 1's choice		c	d		
a	15	0	5	8	10
b	5	10	10	5	0

There are four possible action profiles for player 1 (in  $G^1$  and  $G^2$ ):  $(a, a)$ ,  $(a, b)$ ,  $(b, a)$ ,  $(b, b)$ . Note that

$$\begin{aligned} R_1^2(G^1) &= \{a\}, R_1^1(G^1) = \{a, b\}; \\ R_1^2(G^2) &= \{b\}, R_1^1(G^2) = \{a, b\}. \end{aligned}$$

Clearly, all four action profiles are 1-order rational because  $R_1^1(G^1) = R_1^1(G^2) = \{a, b\}$ , and  $(a, b)$  is the sole action profile that is 2-order rational because  $R_1^2(G^1) \times R_1^2(G^2) = \{(a, b)\}$ . However, the identification problem discussed above applies: player 1 playing  $(a, b)$  does not imply she is 2-order rational because such an action profile may originate from a 1-order but not 2-order rational player. The objective of the ER assumption in Kneeland [13] is to eliminate this possibility. Note that the sole difference between  $G^1$  and  $G^2$  is player 2's payoff, which is the 1-order payoff for player 1. The ER assumption requires a  $k$ -order rational player to not respond to  $k$ -order (or any higher order) payoff changes. Consequently,  $(a, b)$  must originate from a 2-order rational player<sup>2</sup>; this resolves the identification problem described above.

## 4 A Close Examination of the ER Assumption

What does the ER assumption mean? Is it a strong assumption? To answer these questions, we take a close look at decision making of a  $k$ -order rational player.

Facing game  $G$ , a  $k$ -order rational player must choose *one* action. How would she choose it? Let  $\Phi_i^G$  denote the selection process used by the player to choose the final

<sup>2</sup>According to the ER assumption, if the player is 1-order but not 2-order rational, her action is independent of her 1-order payoff, and hence, she must play the same action in both  $G^1$  and  $G^2$ .

action, which is illustrated as follows.

$$G \xrightarrow{\Phi_i^G} a_i.$$

The theory of rationality (refer to, e.g., Bernheim [2] and Pearce [14]) says that this  $k$ -order rational player would perform  $k$  rounds of iterative deletion of strictly dominated strategies. Specifically, given her own payoff  $h_0^i(G)$ , player  $i$  performs 1 round of deletion and gets  $R_i^1(G)$ , which is the set of player  $i$ 's actions that are best responses to her opponents playing actions in  $A_{-i}$ , i.e.,  $h_0^i(G)$  fully determines  $R_i^1(G)$ . Similarly,  $h_1^i(G)$  (i.e.,  $i$ ' opponents' payoffs) fully determines  $R_{-i}^1(G)$  (i.e., the set of  $i$ ' opponents first-order rational actions). Then, given  $h_0^i(G)$  and  $i$ ' opponents playing actions in  $R_{-i}^1(G)$ , player  $i$  performs a 2nd round of deletion and gets  $R_i^2(G)$ . That is,  $[h_0^i, h_1^i(G)]$  fully determines  $R_i^2(G)$ . Furthermore, similar arguments show that  $[h_0^i, h_1^i(G), \dots, h_{k-1}^i(G)]$  fully determines  $R_i^k(G)$ . Let us call this procedure the “deletion step” of player  $i$  choosing one action facing  $G$ . Let  $\Phi_i^{G_d} : G \rightarrow 2^{A_i}$  denote the deletion operation performed by player  $i$  in the deletion step, i.e.,  $\Phi_i^{G_d}(G) \equiv R_i^k(G)$  for a  $k$ -order rational player  $i$ .

However,  $R_i^k(G)$  usually contains multiple actions, which means that the deletion step does not pin down a unique action for player  $i$ . Hence, player  $i$  needs a second step, the “selection step,” to finalize her choice of action among elements in  $R_i^k(G)$ . Let  $\Phi_i^{G_s}$  denote the process used by player  $i$  in the selection step. Thus, the  $\Phi_i^G$  for a  $k$ -order rational player can be decomposed into two steps, as shown below.

$$\Phi_i^G : G \xrightarrow{\Phi_i^{G_d}} R_i^k(G) \xrightarrow{\Phi_i^{G_s}} a_i.$$

From the analysis above, the following proposition is straightforward.

### Proposition 1

$$[h_0^i(G), \dots, h_{k-1}^i(G)] = [h_0^i(G'), \dots, h_{k-1}^i(G')] \implies R_i^k(G) = R_i^k(G') \implies \Phi_i^{G_d} = \Phi_i^{G'_d}.$$

An equivalent way to describe the ER assumption is:

$$[h_0^i(G), \dots, h_{k-1}^i(G)] = [h_0^i(G'), \dots, h_{k-1}^i(G')] \implies \Phi_i^G = \Phi_i^{G'}.$$

Clearly, there is a gap between Proposition 1 and the ER assumption. For the two games  $G$  and  $G'$  with  $[h_0^i(G), \dots, h_{k-1}^i(G)] = [h_0^i(G'), \dots, h_{k-1}^i(G')]$ , Proposition 1 says that  $k$



rounds of iterative deletions would deliver the same  $R_i^k(G) = R_i^k(G')$  for player  $i$ , while the ER assumption requires this agent to select the same *action* in  $G$  and  $G'$ .

Now, consider the following thought experiment. Fix any game  $G$  and let a player play  $G$  twice, i.e., we consider  $G' = G$ . Then, by definition,  $h_k^i(G) = h_k^i(G')$  for any  $k$ , and the ER assumption requires every player to choose the same action on both occasions. Based on this thought experiment, we design laboratory experiments to test the ER assumption. First, we let subjects play each of the following three games – “Meet in New York,” “Matching Penny,” and “Rock-Paper-Scissors” – two times.<sup>3</sup>

Meet in New York			
	$a$	$b$	
$A$	1, 1	0, 0	
$B$	0, 0	1, 1	

Matching Penny		
	$c$	$d$
$C$	1, -1	-1, 1
$D$	-1, 1	1, -1

Rock-Paper-Scissors			
	rock	paper	scissor
rock	0, 0	-5, 5	5, -5
paper	5, -5	0, 0	-5, 5
scissor	-5, 5	5, -5	0, 0

We find that 67.5% of the subjects in rock-paper-scissors, 45% of the subjects in matching penny, and 27.5% of the subjects in meet in New York choose different actions.

Furthermore, we strictly follow the instructions in Kneeland [13], and let the subjects play the ring game (which is used in Kneeland’s identification) two times. If the ER assumption is satisfied, the subjects, regardless of their levels of rationality, should play the same action on both occasions as long as they are in the same position. However, we find that 77.5% of the subjects chose different actions in at least one position; 45% of the subjects chose different actions in at least two positions; 20% of the subjects chose different actions in at least three positions.<sup>4</sup> We describe the aforementioned experiments in detail in Appendix B.

In sum, the ER assumption is not satisfied in the meet in New York, matching penny,

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<sup>3</sup>To exclude the possibility that subjects think of the two games as a rounds of a single game, we 1) did not inform them that they would play the game two times at the beginning of the experiment and 2) did not provide any feedback after the first game.

<sup>4</sup>This result is consistent with Ye [19], who presents an experimental design to identify whether a subject’s exhibited sophistication level is due to his belief in the opponents’ rationality or his ability to finish all the required reasoning steps. Ye [19] reports that 63.1% and 15.1% of subjects remain unidentified when 0 and 1 deviation, respectively, determines the level classification in the ring game. For more details, see Table 4 of Ye [19].

or rock-paper-scissors games. More importantly, the assumption is not satisfied in the ring game, which casts doubt on Kneeland's identification.

## 4.1 An Alternative Interpretation of the ER Assumption

As shown above, one implication of the ER assumption is that, for a fixed game, a player will always choose a fixed rationalizable action whenever she plays the game. Consider the meet in New York game, which has two pure-strategy Nash equilibria,  $(A, a)$  and  $(B, a)$ .

Meet in New York			
		$a$	$b$
$A$	1, 1	0, 0	
$B$	0, 0	1, 1	

Based on the payoffs of the game alone, there is no way for us to tell which equilibrium will be selected by the players. Likewise, there is no way for us to tell which rationalizable actions will be selected by the players. Schelling [16] argues that a “focal point” may determine players' choices. For example, suppose that  $A = a =$  “Grand Central Station” and that  $B = b =$  “Empire State Building.” Then, the players will select  $(A, a)$  if they are New Yorkers, and they will play  $(B, b)$  if they are tourists. However, if player 1 is a New Yorker and player 2 is a tourist, then players' choices remain ambiguous.<sup>5</sup>

Nevertheless, all of the above suggest that the payoffs of a game alone do not determine a rational player's final action; rather, the context of the game (e.g., whether they are tourists) also plays a role. This leads us to provide an alternative interpretation of the ER assumption: it is equivalent to a deterministic selection rule in the selection step. Clearly, our experimental result reveals that most people do not follow a deterministic selection rule when there is no clear focality in one of the available actions. Rather, people select their actions *stochastically*, i.e., they sometimes select “Empire State Building” and sometimes select “Grand Central Station” in the meet in New York game.

What do we mean by the “stochastic selection”? Does a “stochastic process” really exist in the real world? To arrive at an answer, consider a coin toss. Clearly, the outcome

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<sup>5</sup>The two players may argue that since player 1 is a New Yorker, they coordinate to play  $(A, a)$ ; however, they may also argue that since player 2 is a tourist, they coordinate to play  $(B, b)$ .

of a coin toss depends on many factors, for example, the design of the coin, the velocity of air flowing around the coin, the strength of the player who tosses the coin, and how high the coin is tossed. If we know the exact factors that decide the outcome of the coin toss, then the outcome must be deterministic by definition. That is, there exists a deterministic function,  $\Lambda : E^1 \times E^2 \times E^3 \times \dots \rightarrow \{\text{head}, \text{tail}\}$ , where  $E^1, E^2, E^3 \dots$  are the sets of factors that determine the outcome, e.g.,  $E^1$  is the set of coin designs and  $E^2$  is the set of velocities of air flowing around the coin. Unfortunately, as outsiders, we observe only the realized value in  $E^1$  but not those in  $E^2, E^3$ , and so on. Since the realized value in  $E^1$  cannot fully determine the outcome, our prediction is stochastic rather than deterministic, i.e., our prediction is  $\lambda : E^1 \rightarrow \Delta[\{\text{heads}, \text{tails}\}]$ . For example, if the coin has an identical design on both sides, we call it a fair coin, i.e.,  $\lambda(\text{identical design}) = \left[\frac{1}{2}\text{heads} + \frac{1}{2}\text{tails}\right]$ . In sum, when we do not fully know the factors determining the outcome, we use a stochastic process to describe the outcome.<sup>6</sup>

Similarly, the final action chosen by a  $k$ -order rational player in  $G$  depends on many factors, e.g.,  $[h_0^i(G), \dots, h_{k-1}^i(G)]$  (or equivalently,  $R_i^k(G)$ ). However,  $[h_0^i(G), \dots, h_{k-1}^i(G)]$  alone is insufficient to pin down the final action because  $R_i^k(G)$  may contain multiple actions. Thus, players use other factors to pin down the final action, e.g., his habits, his conjectures about his opponent's habits, his mood, and his conjectures about his opponent's mood. That is, in principle, the selection of the final action can be described as a deterministic function,  $\Phi_i : F^1 \times F^2 \times F^3 \times \dots \rightarrow A_i$ , where

$$F^1 = \left\{ [h_0^i(G), \dots, h_{k-1}^i(G)] : \text{all possible game } G \right\}.$$

As a outsider, we observe  $[h_0^i(G), \dots, h_{k-1}^i(G)] \in F^1$  but do not understand and are not fully aware of all other factors that pin down the final action of the  $k$ -order rational player. Therefore, based on the observation on  $F^1$ , we use a function  $\phi_k^i : F^1 \rightarrow \Delta(A_i)$  to describe the outcome. Given this view, it is straightforward to see that the ER assumption (i.e., Assumption 1) is equivalent to the following assumption.

**Assumption 2 (deterministic selection)** *For any  $k$ -order rational player  $i$ ,*

$$\phi_k^i [h_0^i(G), \dots, h_{k-1}^i(G)] \in \{\alpha \in \Delta(A_i) : \alpha(a_i) \in \{0, 1\}, \forall a_i \in A_i\}, \forall G.$$

*That is, player  $i$  uses a Dirac distribution to select an action, given  $[h_0^i(G), \dots, h_{k-1}^i(G)]$ .*

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<sup>6</sup>There are two views of how the stochastic process is determined: the frequentist view and the Bayesian view.

Assumption 2 says that players always use a degenerate (or equivalently, deterministic) rule to select the final action from the set  $R_i^k(G)$ . Clearly, Assumption 2 is critical for Kneeland's identification: if it is not satisfied, a 1-order rational player 1 in the example  $G^1$  and  $G^2$  above would play all  $(a, a)$ ,  $(a, b)$ ,  $(b, a)$ , and  $(b, b)$  with positive probability, and as a result,  $(a, b)$  originates from both 1-order rational players and 2-order rational players—that is, the identification problem remains. Our experiments with Kneeland's ring game, meet in New York, matching penny, and rock-paper-scissors, show that Assumption 2 is not satisfied.<sup>7</sup>

Note that the stochastic selection rule we refer to is different from the randomization induced by payoff indifference due to some non-degenerate belief as discussed in footnote 20 of Kneeland [13]. Even if we exclude such beliefs and eliminate the possibility of payoff indifference, our notion of randomization in action may still exist, which is due to the multiplicity of actions in  $R_i^k(G)$  and the fact that we are not fully aware of all the factors that pin down the final action.

## 5 The Revealed-Rationality Approach

Although higher-order rationality entails the identification problem described above, a player's action in a game indeed provides some information about her level of rationality: playing  $a \notin R_i^k(G)$  implies that the player is not  $k$ -order rational in  $G$ . Thus, we follow Brandenburger, Danieli and Friedenberg [3] and consider an alternative identification strategy:

A player is *not*  $k$ -order **revealed-rational** in game  $G$

if and only if she actually plays an action  $a \notin R_i^k(G)$ .

Equivalently,

A player is  $k$ -order **revealed-rational** in game  $G$

if and only if she *actually* plays actions in  $R_i^k(G)$ . (★★)

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<sup>7</sup>The rock-paper-scissors game has been studied in Cason, Friedman, and Hopkins [5], which also casts doubt on the deterministic-selection assumption. Furthermore, Walker and Wooders [17] conduct an empirical study of Wimbledon games and show that players use mixed strategies instead of pure strategies to select rationalizable actions.

Clearly, we can extend the identification to observed behaviors in multiple games.

A player is  $k$ -order revealed-rational in a set of games  $\mathcal{G}$  (★★★)

if and only if she actually plays actions in  $R_i^k(G)$  for every  $G \in \mathcal{G}$ .

Comparing (★) and (★★), we find that “ $k$ -order rationality” and “ $k$ -order revealed-rationality” are similar, with the former focusing on *all possible behaviors* for players in games and the latter focusing on *all actual behaviors* for players in games.<sup>8</sup>

As discussed above, a player’s final action in a game is determined by her decision making in both the deletion step and the selection step. The *k-order rationality theory* fully describes the decision process in the deletion step, whereas the theory is completely silent regarding the selection step. That is, we attempt to use a player’s final action to infer the order “ $k$ ” in the deletion step *only*; this is the source of the identification problem.

Assuming a particular deterministic selection rule in the selection step (i.e., the ER assumption), Kneeland [13] manages to identify the exact  $k$  rounds of deletion in the deletion step. In contrast, we subsume the selection step into the deletion step; equivalently, we regard the selection step as a possible additional deletion step, and our identified order of revealed rationality summarizes decision making in both the deletion and selection steps.

Does our approach suffer any loss of generality? To us, the answer is no because the ultimate goal of any theory (including  $k$ -order rationality theory) is to predict or describe players’ *behaviors* in games, and our revealed rationality approach achieves this goal better than Kneeland’s approach. For illustration, consider a 1-order but not 2-order rational player who, in any game  $G$ , always performs 1 round of deletion of strictly dominated strategies in the deletion step. However, in the selection step, suppose that this player follows some rule and always chooses an action  $a \in R_i^\infty(G) \subset R_i^1(G)$ . In this case, our identification approach would label this player an  $\infty$ -order revealed-rational player, although the true order of rationality is 1 (i.e., he performs 1 round of deletion in the deletion step). Nevertheless, this order “1” in the deletion step (i.e.,  $R_i^1(G)$ ) is not sufficient to determine the player’s final action in a game, and the final action is always

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<sup>8</sup>However, our approach does not resolve the identification problem. For illustration, consider  $(a, b)$  selected by player 1 in  $G^1$  and  $G^2$  above. Our identification approach simply labels player 1 a 2-order rational player because her behaviors are consistent with  $R_1^2(G^1)$  and  $R_1^2(G^2)$ , although  $(a, b)$  may also originate from a 1-order but not 2-order rational player.

in  $R_i^\infty(G)$ . Hence,  $R_i^\infty(G)$  or “being  $\infty$ -order revealed-rational” is a much better description of the behavior of this player than  $R_i^1(G)$  or “being 1-order rational.” Thus, there is no loss of generality in regarding this player *as if* she were an  $\infty$ -order rational player. This echoes the classical “as-if” view in Friedman [10]: we may never identify the exact decision-making process of agents; however, a model is suitable if the prediction of the model matches the behaviors of the agents.<sup>9</sup>

An alternative interpretation of the revealed-rationality approach is that we provide a conservative estimation of a player’s level of rationality: we say that a player is  $k$ -order rational if and only if her behaviors in game  $G$  do not contradict  $R_i^k(G)$ . The behaviors identify only an *upper bound* on the players exact level of rationality. Hence, the revealed-rationality approach implicitly assumes that the upper bound is equal to the exact level of player’s higher-order rationality, which we call the *upper-bound* assumption. In the following subsection, we discuss the relationship between the ER assumption in Kneeland [13] and our upper-bound assumption.

## 5.1 The ER Assumption vs. the Upper-bound Assumption

Consider the ring games in Kneeland [13], and suppose the ER assumption is satisfied. In the following table, we reproduce Table I of Kneeland [13] which summarizes the predicted actions under different levels of rationality and the ER assumption.

Table I of Kneeland [13]: Predicted actions under rationality and assumptions ER in the 8 games

	Games			
Position	1	2	3	4
Type	$A$ $B$	$A$ $B$	$A$ $B$	$A$ $B$
1-order	$(a, a)(b, b)(c, c)$	$(a, a)(b, b)(c, c)$	$(a, a)(b, b)(c, c)$	$(a, c)$
2-order	$(a, a)(b, b)(c, c)$	$(a, a)(b, b)(c, c)$	$(a, b)$	$(a, c)$
3-order	$(a, a)(b, b)(c, c)$	$(b, a)$	$(a, b)$	$(a, c)$
4-order	$(a, c)$	$(b, a)$	$(a, b)$	$(a, c)$

Precisely, there are 2 ring games, labeled as  $A$  and  $B$ , and 4 positions, labeled as

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<sup>9</sup>This “as-if” view is also discussed in Brandenburger, Danieli and Friedenbergl [3].

1, 2, 3, 4, in each ring game. The two games,  $A$  and  $B$ , are almost the same except for player's payoffs in position 1. Because of the payoff difference, a  $k$ -order rational player has different uniquely rationalizable actions at position  $k'$  ( $\leq k$ ) in  $A$  and  $B$ ; while all actions are rationalizable for this player at position  $k'$  ( $> k$ ) in both games. Given the ER assumption, Table I of Kneeland [13] lists the set of rationalizable actions in both games for players with different levels of rationality. As a result, a player is  $k$ -order rational if she plays different actions in  $A$  and  $B$  at position  $k$ .

However, the upper-bound assumption is also satisfied in these two games, if the ER assumption is satisfied. If a player plays the same action at position  $k + 1$  and different actions at position  $k$ , then  $k$  is the upper bound for the levels of rationality of this player. Indeed, this player is  $k$ -order rational, as implied by the ER assumption. Therefore, the upper-bound assumption is *weaker* than the ER assumption.

Hence, when the ER assumption is satisfied, Kneeland's approach and the revealed-rationality approach deliver the same identification. In general, however, the upper-bound assumption does not imply the ER assumption. In particular, when the ER assumption is not satisfied, Kneeland's approach is completely silent about players' higher-order rationality, while the revealed-rationality still provides a meaningful estimation. Therefore, one way to interpret the revealed-rationality approach is that it generalizes Kneeland's approach to setups in which the ER assumption may not be satisfied.

## 6 The Chain Game

Kneeland [13] argues that bimatrix games cannot be used to identify players' higher-order rationality. Consequently, the ring game is introduced to achieve identification.

Instead of identifying players' higher-order rationality, we identify players' revealed higher-order rationality. Furthermore, we show that bimatrix games are sufficient to achieve the identification. To illustrate the idea, we propose a novel bimatrix game, which we call the *chain game*, described as in game  $G^3$  below.

$$R_1^0(G^3) = \{a_1, a'_1\} \times \{a_2, a'_2\} \times \{a_3, a'_3\}$$

$$R_2^0(G^3) = \{b_1, b'_1\} \times \{b_2, b'_2\} \times \{b_3, b'_3\}$$

$$\begin{array}{c}
\text{player 1's payoff:} \\
\begin{array}{|c|c|} \hline & \\ \hline a_1 & 1 \\ \hline a'_1 & 0 \\ \hline \end{array} + \begin{array}{|c|c|c|} \hline & b_1 & b'_1 \\ \hline a_2 & 2 & 0 \\ \hline a'_2 & 0 & 2 \\ \hline \end{array} + \begin{array}{|c|c|c|} \hline & b_2 & b'_2 \\ \hline a_3 & 1 & 0 \\ \hline a'_3 & 0 & 1 \\ \hline \end{array} \\
\\
\text{player 2's payoff:} \\
\begin{array}{|c|c|} \hline & \\ \hline b_1 & 0 \\ \hline b'_1 & 2 \\ \hline \end{array} + \begin{array}{|c|c|c|} \hline & a_1 & a'_1 \\ \hline b_2 & 1 & 0 \\ \hline b'_2 & 0 & 1 \\ \hline \end{array} + \begin{array}{|c|c|c|} \hline & a_2 & a'_2 \\ \hline b_3 & 2 & 0 \\ \hline b'_3 & 0 & 2 \\ \hline \end{array}
\end{array}$$

Both players choose three-dimensional actions, and a player's payoff is the sum of three parts: the first part is determined by her choice in the 1st dimension of her action alone; the second part by her choice in the 2nd dimension and her opponent's choice in the 1st dimension; the third part by her choice in the 3rd dimension and her opponent's choice in the 2nd dimension.

We design the payoff so that a 1-order rational player has a unique rationalizable choice in the 1st dimension of her action; a 2-order rational player has a unique rationalizable choice in the 2nd dimension; a 3-order rational player has a unique rationalizable choice in the 3rd dimension. Precisely,

$$\begin{aligned}
R_1^1(G^3) &= \{a_1\} \times \{a_2, a'_2\} \times \{a_3, a'_3\}; \\
R_1^2(G^3) &= \{a_1\} \times \{a'_2\} \times \{a_3, a'_3\}; \\
R_1^3(G^3) &= \{a_1\} \times \{a'_2\} \times \{a_3\};
\end{aligned}$$

We can identify the revealed higher-order rationality as follows:

player 1 is 1-order revealed-rational if and only if she plays  $a_1$ ;

player 1 is 2-order revealed-rational if and only if she plays  $(a_1, a'_2)$ ;

player 1 is 3-order revealed-rational if and only if she plays  $(a_1, a'_2, a_3)$ .

One way to interpret the chain game is that we merge two  $n$ -player ring games into a 2-player chain game. For example,  $G^3$  corresponds to the two ring games,  $G^4$  and  $G^5$ , in Figure 1. For every  $n \in \{1, 2, 3\}$ , if we assign a player to the position of "player  $n$ " in ring games  $G^4$  and  $G^5$ , her behaviors in the games would reveal whether she is  $n$ -order rational. The chain game  $G^3$  can be considered to be a combination of  $G^4$  and  $G^5$ , or more



precisely, assigning player 1 in  $G^3$  to the positions of “player 1” and “player 3” in  $G^4$  and “player 2” in  $G^5$  simultaneously; and assigning player 2 in  $G^3$  to the positions of “player 2” in  $G^4$  and “player 1” and “player 3” in  $G^5$  simultaneously. Furthermore, the payoffs of the players in  $G^3$  become the sum of the payoffs of their positions in  $G^4$  and  $G^5$ .

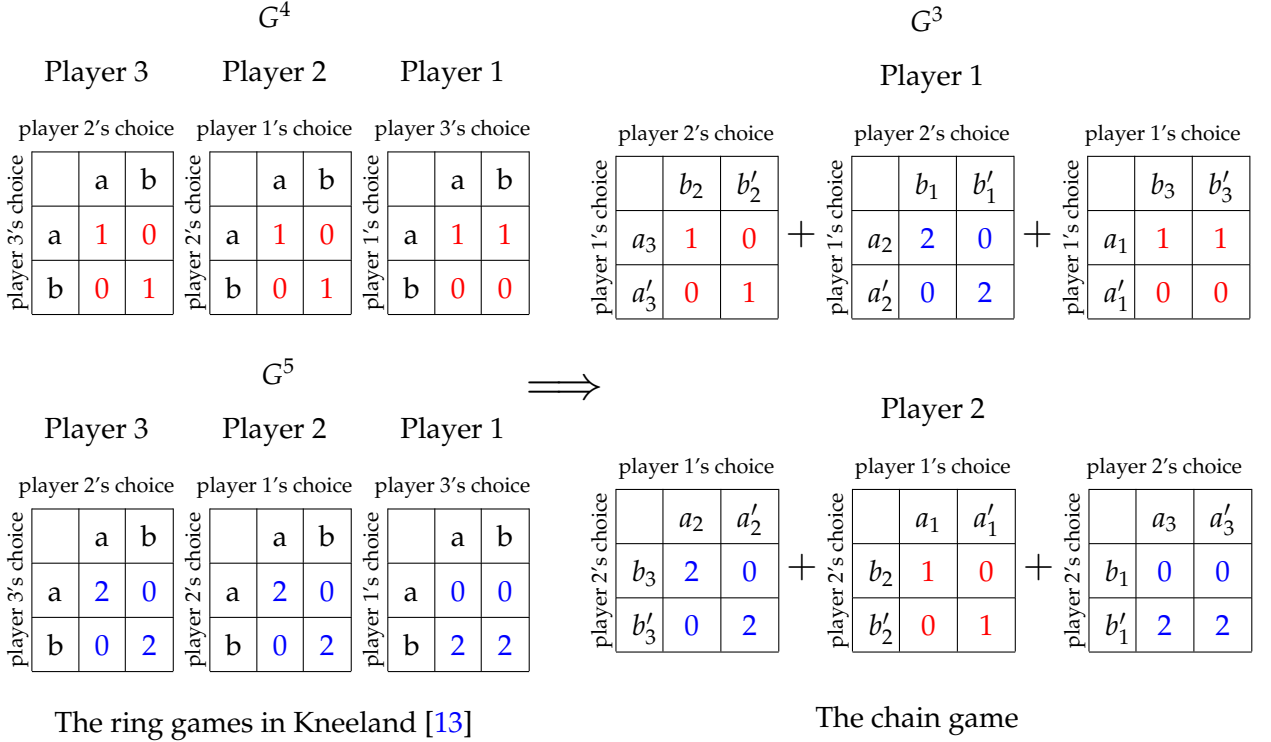


Figure 1: Transformation of 3-person ring games to a 2-person chain game

Similarly, we transform the ring games used in Kneeland’s [13] experiments to the chain games and conduct experiments to identify players’ revealed higher-order rationality. Our experimental results show that 96% of the participants are 1-order rational, 63% are 2-order rational, 21% are 3-order rational, and 10% are 4-order rational (refer to Appendix A).

Compared with the ring game, the chain game has two advantages. First, the experiments using the chain game require fewer participants, and hence the chain game is more efficient. Second, the chain game identifies all levels of (revealed) higher-order rationality *simultaneously*, whereas in ring games, we need to rotate the players’ positions multiple times, and the players must play multiple rounds of the same game (albeit in

Table 1: Identification results

	1-order	2-order	3-order	4-order
Identification in Kneeland [13]	93%	71%	44%	22%
Our experimental result	96%	63%	21%	10%

different positions).<sup>10</sup> Consequently, participants in ring games may learn how to play the game as the experiments continue<sup>11</sup>; this *learning effect* may thus distort the identification outcome.<sup>12</sup> Compared with Kneeland’s data, our experimental outcome appears to suggest that the learning effect in Kneeland [13] is significant.<sup>13</sup>

<sup>10</sup>Note that the notion of “ $k$ -order” rationality considered in Kneeland [13] is defined in static (rather than dynamic) games. For notions of rationality in dynamic games, refer to, e.g., Battigalli [1] and Pearce [14]. The chain game identifies the purely static higher-order rationality, whereas Kneeland’s identification involves a dynamic flavor.

<sup>11</sup>Weber [18] shows that subjects in the lab can learn from introspection when they play a game repeatedly, even if no feedback is provided.

<sup>12</sup>For example, suppose that a 1-order but not 2-order rational player is first assigned to the position of “player 1” in  $G^4$ , and she deletes action  $b$  after performing 1 round of deletion of strictly dominated strategies. Furthermore, suppose this player is later assigned to the position of “player 2” in  $G^4$ . Then, her *prior experience* and one (additional) round of deletion of strictly dominated strategy would lead her to play only  $a$ ; such a behavior would mislead us to believe she is a 2-order rational player.

<sup>13</sup>For a  $k$ -order but not  $(k + 1)$ -order rational subject, we use  $k$  to label this subject. Then, the identified distribution of the random variable  $k$  in Kneeland [13] has the PDF:  $\Pr[k = 0] = 7\%$ ,  $\Pr[k = 1] = 22\%$ ,  $\Pr[k = 2] = 27\%$ ,  $\Pr[k = 3] = 22\%$ , and  $\Pr[k \geq 4] = 22\%$ . We can calculate our distribution similarly. The CDFs of the two distributions are listed below.

	$\Pr[k \leq 0]$	$\Pr[k \leq 1]$	$\Pr[k \leq 2]$	$\Pr[k \leq 3]$
Identification in Kneeland [13]	7%	29%	56%	78%
Our experimental result	4%	37%	79%	90%

That is, Kneeland’s distribution has (approximate) first-order stochastic dominance over ours, which may be caused by the learning effect.

## 7 Concluding Remarks

Kneeland [13] argues that one major merit of its identification approach is that it requires a considerably weaker assumption (i.e., the ER assumption) than those imposed in previous papers. In this paper, we impose an even weaker assumption and furthermore, our identification approach can be implemented using the chain game, which is simpler and more efficient than Kneeland's [13] ring game.

Finally, we discuss two potential critiques, which are shared by this paper and Kneeland [13]. First, to make our identification useful, both of the two papers implicitly assume that players' levels of rationality do not vary across games. If they do, the identified level in some game does not provide the information on other games. However, Georganas, Healy and Weber [11] show that distribution of level-k types in the population might be constant across games, though individual players' levels may vary across game. Thus, alternatively, the identification in Kneeland [13] and this paper can be viewed as identifying a distribution of level-k types in the population.

To see the 2nd potential critique, consider the following scenario.

$$\begin{aligned} R_1^1(G') &= R_1^1(G'') = \{a, b\}; \\ R_1^2(G') &= R_1^2(G'') = \{a\}. \end{aligned}$$

Suppose a 1-order but not 2-order rational player 1 who always selects  $a$  to play in  $G'$  but plays both  $a$  and  $b$  in  $G''$ . If we conduct experiments only on  $G'$ , we would incorrectly conclude that the player is (revealed) 2-order rational.

The critique is equivalent to stating that the sampling of only  $G'$  is not sufficient to identify the players' true order of rationality.<sup>14</sup> However, one implicit assumption that is imposed by the revealed preference theory, by Kneeland [13] and by us is as follows:

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<sup>14</sup>This critique also applies to the revealed preference theory. To see this, we use  $C(B, \succeq)$  to denote the set of alternatives that an agent with preference  $\succeq$  actually chooses in budget set  $B$ . Suppose  $\alpha, \beta \in B$ ,  $\alpha \neq \beta$  and  $\alpha \sim \beta \succ \gamma$  for every  $\gamma \in B \setminus \{\alpha, \beta\}$ . To obtain the full set of  $C(B, \succeq)$ , we need to observe multiple occasions of "the agent choosing an alternative in  $B$ " because the agent selects only *one* optimal alternative in  $B$  for each occasion. Consider a sequence of occasions such that the agent chooses  $\alpha$  in the first  $k$  occasions but changes to  $\beta$  in all other occasions. In reality, if we have only *finite*, say  $n$ , observations with  $n \leq k$ , we would incorrectly conclude that  $\alpha$  is (revealed) strictly preferred to  $\beta$ . That is, the sampling of  $n (\leq k)$  observations is not sufficient to identify the true preference.

we have a good sampling of observations, so that the observed behaviors of the agents are good enough representation of their behaviors in unobserved occasions.

Given this assumption, both Kneeland [13] and we provide a method for identifying the players' higher-order rationality.

Such a critique may also apply to the statistical sampling. However, statistical results (e.g., the central limit theorem) address the critique by providing a way to judge whether a statistical sampling is good. In contrast, we do not have a theory to help us judge whether an experimental sampling is good. Nevertheless, finding such a theory is clearly beyond the scope of this paper.

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## Appendix A – Experimental Design and Findings

In this section, we report our experimental design and findings for two sets of treatments. The first treatment was designed and conducted to identify the revealed higher-order rationality using the chain game. The second treatment was designed and conducted to test whether or not individuals use the deterministic choice rule in various games including the rock-paper-scissor game, the matching penny game, the pure-coordination game, and the ring game of Kneeland [13].

The experiments were conducted in English using z-tree (Fischbacher [9]) at the Hong Kong University of Science and Technology Experimental Laboratory. For the first treatment, three sessions were conducted, each of which involved ten to thirteen pairs of individuals. For the second treatment, two sessions were conducted and each session involved twenty individuals. In total, 108 subjects participated in the five sessions. On average, each session lasted for 1 hour. The average payment (including the HKD40 show-up fee) was HKD143 ( $\approx$  USD18.4). Subjects had no prior experience in our experiments and were recruited from the undergraduate / graduate population of the university. Upon arrival at the lab, subjects were instructed to sit at separate computer terminals. Each was provided with a copy of the experimental instructions (refer to Appendices C and D for Treatments 1 and 2, respectively). Instructions were read aloud and supplemented by slide illustrations.

### A.1 Treatment 1

We conducted an experiment that consists of two parts. Two participants were anonymously matched, and in Part I, they played the classical rock-paper-scissor game three times. In part II, the two participants played two chain games transformed from the ring games in Kneeland [13, G1 on Page 2072 and G2 on Page 2,073]. More precisely,  $G^6$  below describes the first chain game; the second chain game, denoted by  $G^7$ , then results from switching players' payoffs (or equivalently, roles) in  $G^6$ .<sup>15</sup> Throughout the experiment, the participants could not observe their opponents' actions.

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<sup>15</sup>Member 1's and Member 2's payoff matrix in  $G^6$  describes the first and the second ring games in Kneeland [13], respectively. Each participant in Kneeland's experiment plays *both* ring games; therefore, we allowed participants in our experiment to play both chain games.

$G^6$

2's choice					2's choice					2's choice					2's choice							
1's choice		$b_1$	$b_2$	$b_3$	+	1's choice		$c_1$	$c_2$	$c_3$	+	1's choice		$d_1$	$d_2$	$d_3$	+	1's choice		$a_1$	$a_2$	$a_3$
	$A_1$	8	20	12			$B_1$	14	18	4			$C_1$	20	14	8			$D_1$	12	16	14
	$A_2$	0	8	16			$B_2$	20	8	14			$C_2$	16	2	18			$D_2$	8	12	10
	$A_3$	18	12	6			$B_3$	0	16	18			$C_3$	0	16	16			$D_3$	6	10	8

Member 1's earnings

1's choice					1's choice					1's choice					1's choice							
2's choice		$B_1$	$B_2$	$B_3$	+	2's choice		$C_1$	$C_2$	$C_3$	+	2's choice		$D_1$	$D_2$	$D_3$	+	2's choice		$A_1$	$A_2$	$A_3$
	$a_1$	8	20	12			$b_1$	14	18	4			$c_1$	20	14	8			$d_1$	8	12	10
	$a_2$	0	8	16			$b_2$	20	8	14			$c_2$	16	2	18			$d_2$	6	10	8
	$a_3$	18	12	6			$b_3$	0	16	18			$c_3$	0	16	16			$d_3$	12	16	14

Member 2's earnings

The  $k$ -order rationalizable actions of the two chain games in Part II are described in Table 2.

Table 2: Predicted action profiles from  $\{G^6, G^7\}$

	Member 1		Member 2	
Type	$G^6$	$G^7$	$G^6$	$G^7$
1-order	$D_1$	$D_3$	$d_3$	$d_1$
2-order	$(C_2, D_1)$	$(C_1, D_3)$	$(c_1, d_3)$	$(c_2, d_1)$
3-order	$(B_2, C_2, D_1)$	$(B_1, C_1, D_3)$	$(b_1, c_1, d_3)$	$(b_2, c_2, d_1)$
4-order	$(A_3, B_2, C_2, D_1)$	$(A_1, B_1, C_1, D_3)$	$(a_1, b_1, c_1, d_3)$	$(a_3, b_2, c_2, d_1)$

The experimental outcome is reported in Appendix B and it is also summarized in Table 3 below. The first row presents the player's revealed higher-order rationality identified by our data from Part II. The second and third rows present the percentage of subjects who choose the same action and different actions, respectively, in the three rock-paper-scissor games in Part I, conditional on being categorized as each order in Part II.



For example, column (1) of the table tells us that 96% of the subjects were identified as 1-order rational in Part II, and among the 1-order rational players, 26% chose the same action, and 74% chose different actions in the three rock-paper-scissor games in Part I.

Table 3: Summary of Experimental Outcome

		Revealed-rationality based on Part II					
		(1)	(2)	(3)	(4)	(5)	(6)
		1-order	2-order	3-order	1-3 orders	4-order	Total
% of subjects		96%	63%	21%	90%	10%	100%
Actions in Part I	Same action	26%	21%	21%	26%	29%	26%
	Different actions	74%	79%	79%	74%	71%	74%

If the deterministic-selection assumption (i.e., Assumption 2) holds, the participants would play the same action for all three rock-paper-scissor games. Hence, the result in Table 3 implies that 74% of the data violate Assumption 2.

Note that the ER assumption applies solely to agents with finite-order rationality. Hence, we focus on the 90% of the participants (i.e., those who are not 4-order rational) to test the ER assumption (Column (4) of Table 3).<sup>16</sup> Because the payoffs of the three rock-paper-scissor games remain unchanged, the players'  $k$ -order payoffs are exactly the same for any  $k$ . Hence, conditional on players not being 4-order rational, 74% of our data violate the ER assumption. Consequently, conditional on the players being finite-order rational, at least 66% ( $\approx 90\% \times 74\%$ ) of our data violate the ER assumption.<sup>17</sup>

## A.2 Treatment 2

We conducted an experiment that consists of four sections. At the beginning of Sections 1 and 3, four participants were anonymously matched to play eight ring games of Kneeland [13]. The orders of the eight games were randomly determined. We followed the

<sup>16</sup>Subjects who are identified as being 4-order rational in our experiments may also be  $\infty$ -order rational. Thus, we excluded them when we tested the ER assumption.

<sup>17</sup>We also examined our Part II data ( $G^6$  and  $G^7$ ) to determine how many subjects violate the ER assumption. Of our data, 28% violate the ER assumption, compared with 16% of the data that violate the ER assumption in Kneeland [13].

same procedure used in Kneeland [13].<sup>18</sup> At the beginning of Sections 2 and 4, two participants were anonymously matched to play the following three 2-player games: Meet in New York, Matching Pennies, and Rock-Paper-Scissor games. The orders of the three games were randomly determined. Throughout the experiment, we did not provide any information feedback to make sure that the participants cannot observe their opponents' actions. At the beginning of the experiment, we only revealed that the experiment consists of four sections but the subjects were informed about which game they play in each section only when they begin the new section.

We find that 67.5% of subjects in "rock-paper-scissor", 45% of subjects in "matching penny", and 27.5% of subjects in the pure-coordination game chose different actions, which implies violation of the ER assumption. If Kneeland's ER assumption is satisfied, subjects, regardless of their levels of rationality, should play the same action in the two rounds as long as they are in the same position. However, we indeed found

- 77.5% of subjects chose different actions at least in one position,
- 45% of subjects chose different actions at least in two positions, and
- 20% of subjects chose different actions at least in three positions.

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<sup>18</sup>The instructions of Kneeland [13] were adopted to conduct the ring games for Sections 1 and 3.

# Appendix B.1 – Experimental Data and Subject Classification for Treatment 1

Session	Subject	Role	Task 1 - RPS Game			Task 2 - Game 1				Task 2 - Game 2				Revealed Order	ER Assumption (Task 2 only)
			Game 1	Game 2	Game 3	Same Action?	Action 1	Action 2	Action 3	Action 4	Action 1	Action 2	Action 3		
1	1	1	1	1	2	0	3	1	2	1	1	1	3	2	1
1	2	2	3	3	3	1	1	2	1	3	1	2	1	2	0
1	3	1	1	3	2	0	1	2	1	1	1	2	3	1	0
1	4	1	1	1	1	1	3	2	2	1	1	1	3	3	1
1	5	2	1	1	3	0	3	1	1	3	1	2	1	3	1
1	6	2	1	3	1	0	1	2	1	3	1	2	1	1	0
1	7	1	3	1	1	0	1	2	2	1	1	2	3	2	0
1	8	2	3	2	1	0	1	2	1	3	1	2	1	2	0
1	9	2	2	3	3	0	1	2	1	3	1	2	1	2	0
1	10	2	2	1	2	0	1	1	1	3	3	1	2	2	1
1	11	1	1	1	3	0	1	2	2	1	1	2	1	3	0
1	12	1	2	1	2	0	1	2	2	1	1	1	3	4	N/A
1	13	2	1	1	1	1	1	2	1	3	1	2	1	1	0
1	14	1	1	1	1	1	1	2	1	1	1	2	3	1	0
1	15	1	3	2	3	0	3	2	2	1	1	1	3	3	1
1	16	2	2	2	2	1	3	3	1	3	1	2	1	2	1
1	17	2	2	1	1	0	1	1	1	3	1	2	1	3	0
1	18	2	1	1	1	1	1	2	1	3	3	2	1	2	1
1	19	1	1	3	3	0	1	2	2	1	1	2	3	2	0
1	20	1	2	3	3	0	3	2	2	1	1	1	3	3	1
1	21	1	1	3	2	0	1	2	2	1	1	2	3	2	0
1	22	2	1	2	1	0	1	1	1	3	3	2	1	4	N/A
1	23	1	3	2	3	0	3	2	2	1	1	2	3	2	1
1	24	2	1	2	2	0	1	2	1	3	1	2	1	1	0
1	25	1	2	2	3	0	1	3	3	3	1	2	3	0	1
1	26	2	2	2	2	0	1	2	1	3	1	2	1	2	0
2	27	1	2	1	2	0	1	2	2	1	1	1	3	4	N/A
2	28	2	2	3	3	0	1	2	1	3	1	2	1	1	0
2	29	1	2	2	3	0	3	2	1	1	1	2	3	1	1
2	30	2	1	3	1	0	1	2	1	3	1	2	1	2	0
2	31	1	2	2	2	1	1	2	1	1	1	2	3	1	0
2	32	2	3	1	3	0	1	1	1	3	3	2	1	4	N/A
2	33	1	3	2	2	0	1	2	1	1	1	1	3	1	1

2	34	2	1	3	2	0	1	2	1	3	2	1	1	2	1	1	0
2	35	1	2	3	2	0	1	2	1	1	2	1	1	2	1	3	0
2	36	2	3	2	2	0	1	2	1	3	2	1	1	2	1	3	0
2	37	1	2	1	1	0	1	2	2	1	2	1	1	2	1	3	0
2	38	2	3	3	3	1	1	2	1	3	2	1	1	2	1	3	0
2	39	1	3	3	3	1	1	2	2	1	2	1	1	2	1	3	N/A
2	40	2	2	2	3	0	2	2	2	3	2	1	3	1	3	4	1
2	41	1	2	1	1	1	1	2	1	1	2	1	1	1	1	3	N/A
2	42	2	2	2	2	1	1	1	1	3	1	1	1	1	1	4	1
2	43	1	2	2	2	1	1	2	1	1	2	1	1	1	3	1	0
2	44	2	1	1	3	0	1	2	1	3	2	1	1	1	1	1	0
2	45	1	3	3	3	1	1	2	1	1	2	1	1	1	3	1	0
2	46	2	3	1	3	0	1	2	1	3	2	1	1	2	1	2	0
2	47	1	1	1	1	1	1	2	1	1	2	1	1	1	3	1	0
2	48	2	1	1	3	0	1	2	1	3	1	1	2	1	1	2	1
3	49	1	2	2	2	1	3	2	2	2	2	1	1	2	3	0	1
3	50	2	2	2	2	1	1	2	1	3	2	2	2	2	1	2	1
3	51	1	1	3	3	0	1	2	2	1	2	1	1	1	3	2	0
3	52	2	3	2	3	0	1	2	1	3	2	1	2	2	1	2	0
3	53	1	2	3	1	0	1	2	2	1	2	1	1	1	3	2	0
3	54	2	3	2	2	0	1	2	1	3	2	1	1	2	1	2	0
3	55	1	2	1	2	0	3	2	2	1	2	1	1	1	3	3	0
3	56	2	3	3	3	1	2	3	1	3	3	1	2	2	1	2	1
3	57	1	1	2	2	0	1	2	1	1	2	1	1	1	3	1	0
3	58	2	3	2	2	0	1	1	1	3	2	2	2	2	1	4	N/A
3	59	1	2	3	1	0	1	2	1	1	2	1	1	1	1	0	0
3	60	2	2	1	2	0	3	1	1	3	2	2	2	1	3	3	0
3	61	1	3	1	3	0	3	1	2	1	1	1	1	1	3	2	0
3	62	2	3	1	3	0	1	2	1	3	2	2	2	2	1	2	0
3	63	1	2	1	1	0	1	1	2	1	1	1	1	1	3	2	0
3	64	2	3	3	1	0	1	1	2	3	1	2	2	2	1	1	1
3	65	1	1	1	1	0	1	2	1	1	2	1	1	1	3	1	0
3	66	2	2	2	2	1	1	2	2	3	2	2	2	2	1	1	0
3	67	1	2	3	3	0	1	2	1	1	2	1	1	1	3	1	0
3	68	2	2	1	3	0	3	2	1	3	2	2	2	2	1	2	0

Role: 1 = Member 1, 2 = Member 2

Task 1 - RPS Game: 1 = Rock, 2 = Paper, 3 = Scissor; Same action: 1 = Yes (same action choice in all three games), 0 = No (different action choices across games)

Task 2 - Action 1 (Member 1 / Member 2): 1 =  $A_1 / a_1$ , 2 =  $A_2 / a_2$ , 3 =  $A_3 / a_3$ ; Action 2 (Member 1 / Member 2): 1 =  $B_1 / b_1$ , 2 =  $B_2 / b_2$ , 3 =  $B_3 / b_3$ ;

Action 3 (Member 1 / Member 2): 1 =  $C_1 / c_1$ , 2 =  $C_2 / c_2$ , 3 =  $C_3 / c_3$ ; Action 4 (Member 1 / Member 2): 1 =  $D_1 / d_1$ , 2 =  $D_2 / d_2$ , 3 =  $D_3 / d_3$ ;

Revealed Order: 0 = 0-order, 1 = 1-order, 2 = 2-order, 3 = 3-order, 4 = 4-order

ER Assumption: 0 = Satisfied, 1 = Violated, N/A: Not Applicable

## Appendix B.2 – Experimental Data and Violation of ER for Treatment 2

Session	Subject	Two-times Repeated Ring Game 1								Two-times Repeated Ring Game 2				Total # of Violations of ER	
		Position 1	Position 2	Position 3	Position 4	Position 1	Position 2	Position 3	Position 4	Position 1	Position 2	Position 3	Position 4	Total # of Violations of ER	
1	1	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	1	0
1	2	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	1	1
1	3	Yes	Yes	Yes	Yes	Yes	No	No	Yes	No	No	No	Yes	2	1
1	4	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	0	1
1	5	Yes	Yes	Yes	Yes	No	No	No	Yes	No	No	Yes	No	3	1
1	6	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	0	2
1	7	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	0	0
1	8	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	No	No	Yes	Yes	1	2
1	9	Yes	Yes	Yes	Yes	No	Yes	No	Yes	Yes	No	No	Yes	2	1
1	10	No	Yes	Yes	Yes	Yes	No	No	Yes	No	No	No	No	3	2
1	11	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	No	No	No	Yes	1	1
1	12	No	Yes	Yes	Yes	No	No	Yes	Yes	Yes	Yes	Yes	Yes	3	2
1	13	Yes	Yes	Yes	Yes	No	Yes	No	Yes	No	No	No	No	2	2
1	14	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	0	2
1	15	Yes	Yes	Yes	Yes	No	No	Yes	Yes	No	No	Yes	Yes	2	0
1	16	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	1	1
1	17	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	1	0
1	18	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	0	2
1	19	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	0	2
1	20	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	No	No	No	Yes	1	1

2	2	1	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	1	Yes	No	Yes	1
2	2	2	No	Yes	Yes	Yes	No	No	Yes	Yes	3	Yes	Yes	Yes	0
2	2	3	No	Yes	Yes	Yes	No	No	Yes	Yes	3	No	No	No	3
2	2	4	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	1	Yes	No	No	2
2	2	5	Yes	Yes	Yes	Yes	No	No	Yes	Yes	2	Yes	No	No	2
2	2	6	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	1	No	No	No	3
2	2	7	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	1	Yes	No	No	2
2	2	8	No	Yes	Yes	Yes	No	No	Yes	Yes	3	No	No	No	3
2	2	9	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	0	Yes	No	Yes	1
2	2	10	Yes	Yes	Yes	Yes	No	No	Yes	Yes	2	Yes	Yes	Yes	0
2	2	11	Yes	Yes	Yes	Yes	No	Yes	No	Yes	2	Yes	Yes	No	1
2	2	12	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	0	No	Yes	No	2
2	2	13	Yes	Yes	Yes	Yes	Yes	No	No	Yes	2	No	No	No	3
2	2	14	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	1	No	No	Yes	2
2	2	15	Yes	Yes	Yes	Yes	No	No	Yes	Yes	2	No	No	Yes	2
2	2	16	Yes	Yes	Yes	Yes	No	No	No	Yes	3	Yes	Yes	Yes	0
2	2	17	Yes	No	No	No	Yes	No	Yes	Yes	4	No	No	No	3
2	2	18	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	0	Yes	Yes	Yes	0
2	2	19	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	1	Yes	No	Yes	1
2	2	20	Yes	Yes	Yes	Yes	No	No	Yes	Yes	2	Yes	Yes	No	1

Yes = same action choice in the given position of the two-times repeated game, No = different action choices

# Appendix C – Experimental Instructions for Treatment 1

## INSTRUCTION

Welcome to the experiment. This experiment studies decision making between two individuals. In the following hour or less, you will participate in decision making for **two tasks** (Tasks 1 and 2). Please read the instructions below carefully; the cash payment you will receive at the end of the experiment depends on how you make your decisions according to these instructions. Communication of any kinds with any other participants will not be allowed.

## Your Cash Payment

Your total cash payment at the end of the experiment will be the **sum** of the number of tokens that you will earn from the **two tasks**, translated into HKD with the exchange rate of 1 Token = 0.8 HKD, plus a 40 HKD show-up fee.

## Your Group

There are 20 participants in today's session. Half of the participants will be randomly assigned the role of Member 1 and the other half the role of Member 2. Your role will remain fixed throughout the experiment. Prior to the first task, one Member 1 will be randomly and anonymously paired with one Member 2 to form a group of two. The two members in a group make decisions that will affect their earnings from both tasks. Note that no information feedback about the other member's choices will be provided in the entire course of decision makings. You will not be told the identity of the participant with whom you are matched, nor will that participant be told your identity—even at the end of the experiment.

## Task 1: Your Decision

At the beginning of Task 1, you will be randomly matched with a participant in the other role in this room to play the following "rock-paper-scissor" game for *three* times. For *each* game, both members of your group will independently choose one of the three actions, **rock, paper, and scissor**.

		Member 1's earning		
		2's choice		
1's choice		rock	paper	scissor
	rock	0	-5	5
	paper	5	0	-5
	scissor	-5	5	0

		Member 2's earning		
		1's choice		
2's choice		rock	paper	scissor
	rock	0	-5	5
	paper	5	0	-5
	scissor	-5	5	0

Your earnings will depend on the combination of your action and your opponent's action. These earning possibilities are represented in the table above. In your earning table, your action will determine the row and your opponent's action will determine the column. For example, if you choose "rock" and your opponent chooses "scissor", you would earn 5 tokens. If instead your opponent chooses "paper", you would earn -5. (Note that because of the show-up fee HKD 40, you will always have **positive earnings** at the end of the experiment.)

You will be asked to play the game three times, and you will *not* observe your opponent's choices throughout the process. Your earnings from Task 1 is the sum of your earnings from the three games.

### Task 2: Your Decision

You and your opponent will be asked to play *two* games of the following form — the *only* difference between the following game and the actual games that you will play is the payoffs.

Member 1's set of actions:  $\{A_1, A_2, A_3\} \times \{B_1, B_2, B_3\} \times \{C_1, C_2, C_3\} \times \{D_1, D_2, D_3\}$ ;

Member 2's set of actions:  $\{a_1, a_2, a_3\} \times \{b_1, b_2, b_3\} \times \{c_1, c_2, c_3\} \times \{d_1, d_2, d_3\}$ .

		2's choice		
1's choice		$b_1$	$b_2$	$b_3$
	$A_1$	10	4	16
	$A_2$	20	8	0
	$A_3$	4	18	12

+

		2's choice		
1's choice		$c_1$	$c_2$	$c_3$
	$B_1$	12	16	4
	$B_2$	0	12	8
	$B_3$	4	4	20

+

		2's choice		
1's choice		$d_1$	$d_2$	$d_3$
	$C_1$	20	12	8
	$C_2$	6	8	18
	$C_3$	0	16	4

+

		2's choice		
1's choice		$a_1$	$a_2$	$a_3$
	$D_1$	10	12	8
	$D_2$	6	20	18
	$D_3$	16	4	0

Figure 1: Member 1's earnings

		1's choice		
2's choice		$B_1$	$B_2$	$B_3$
	$a_1$	6	20	18
	$a_2$	16	4	0
	$a_3$	10	12	8

+

		1's choice		
2's choice		$C_1$	$C_2$	$C_3$
	$b_1$	20	12	8
	$b_2$	6	8	18
	$b_3$	0	16	4

+

		1's choice		
2's choice		$D_1$	$D_2$	$D_3$
	$c_1$	10	4	16
	$c_2$	20	8	0
	$c_3$	4	18	12

+

		1's choice		
2's choice		$A_1$	$A_2$	$A_3$
	$d_1$	12	16	4
	$d_2$	0	12	8
	$d_3$	4	4	20

Figure 2: Member 2's earnings



Member 1 and Member 2 simultaneously and independently make decisions. Member 1 chooses one action out of a set of 12 actions described by  $\{A_1, A_2, A_3\} \times \{B_1, B_2, B_3\} \times \{C_1, C_2, C_3\} \times \{D_1, D_2, D_3\}$ , i.e., Member 1 chooses an action  $(A, B, C, D)$  with  $A \in \{A_1, A_2, A_3\}$ ,  $B \in \{B_1, B_2, B_3\}$ ,  $C \in \{C_1, C_2, C_3\}$ , and  $D \in \{D_1, D_2, D_3\}$ .

Similarly, Member 2 chooses an action  $(a, b, c, d)$  with  $a \in \{a_1, a_2, a_3\}$ ,  $b \in \{b_1, b_2, b_3\}$ ,  $c \in \{c_1, c_2, c_3\}$ , and  $d \in \{d_1, d_2, d_3\}$ .

The tables in Figure 1 describe the earnings of Member 1 in the game. Given  $(A, B, C, D)$  chosen by Member 1 and  $(a, b, c, d)$  chosen by Member 2, the earnings of Member 1 consists of four parts:  $(A, b)$  in the first table identifies the first part;  $(B, c)$  in the second table identifies the second part;  $(C, d)$  in the third table identifies the third part;  $(D, a)$  in the fourth table identifies the fourth part. Member 1's final earning is the *sum* of the four parts.

Similarly, the tables in Figure 2 describe the earnings of Member 2 in the game. Given  $(A, B, C, D)$  chosen by Member 1 and  $(a, b, c, d)$  chosen by Member 2, the earnings of Member 2 consists of four parts:  $(a, B)$  in the first table identifies the first part;  $(b, C)$  in the second table identifies the second part;  $(c, D)$  in the third table identifies the third part;  $(d, A)$  in the fourth table identifies the fourth part. Member 2's final earning is the *sum* of the four parts.

For example, suppose that Member 1's choice is  $(A_1, B_1, C_3, D_1)$  and Member 2's choice is  $(a_1, b_2, c_1, d_1)$ . Then, Member 1's earnings become 4 (the first table) + 12 (the second table) + 0 (the third table) + 10 (the forth table) = 26. Member 2's earnings become 6 (the first table) + 18 (the second table) + 10 (the third table) + 12 (the forth table) = 46.

You will play *two* games of the form as above sequentially while you will *not* observe your opponent's choices throughout the process. Your earnings from Task 2 is the sum of your earnings from these two games. You will be required to spend at least 300 seconds on each game. You may spend more time on the games if you wish.

### **Administration**

Your decisions as well as your monetary payment will be kept confidential. Remember that you have to make your decisions entirely on your own; please do not discuss your decisions with any other participants.

Upon finishing the experiment, you will receive your cash payment. You will be asked to sign your name to acknowledge your receipt of the payment. You are then free to leave.

If you have any question, please raise your hand now. We will answer your question individually.

To ensure your comprehension of the instructions, we will provide you with a quiz in the next page. We will go through the quiz after you answer it on your own. Note that the quiz is also part of instructions, and your answers in the quiz do not affect your earnings from today's session.

### Quiz

2's choice					2's choice					2's choice					2's choice							
1's choice		$b_1$	$b_2$	$b_3$	+	1's choice		$c_1$	$c_2$	$c_3$	+	1's choice		$d_1$	$d_2$	$d_3$	+	1's choice		$a_1$	$a_2$	$a_3$
	$A_1$	10	4	16			$B_1$	12	16	4			$C_1$	20	12	8			$D_1$	10	12	8
	$A_2$	20	8	0			$B_2$	0	12	8			$C_2$	6	8	18			$D_2$	6	20	18
	$A_3$	4	18	12			$B_3$	4	4	20			$C_3$	0	16	4			$D_3$	16	4	0

Figure 1: Member 1's earnings

1's choice					1's choice					1's choice					1's choice							
2's choice		$B_1$	$B_2$	$B_3$	+	2's choice		$C_1$	$C_2$	$C_3$	+	2's choice		$D_1$	$D_2$	$D_3$	+	2's choice		$A_1$	$A_2$	$A_3$
	$a_1$	6	20	18			$b_1$	20	12	8			$c_1$	10	4	16			$d_1$	12	16	4
	$a_2$	16	4	0			$b_2$	6	8	18			$c_2$	20	8	0			$d_2$	0	12	8
	$a_3$	10	12	8			$b_3$	0	16	4			$c_3$	4	18	12			$d_3$	4	4	20

Figure 2: Member 2's earnings

Consider the game above.

1. Suppose that Member 1's choice is  $(A_2, B_1, C_1, D_3)$  and Member 2's choice is  $(a_1, b_3, c_2, d_2)$ .

1.1 Suppose that you are Member 1. What is **your** earning from

- (a) the first table? (Your answer: \_\_\_\_\_)
- (b) the second table? (Your answer: \_\_\_\_\_)
- (c) the third table? (Your answer: \_\_\_\_\_)
- (d) the forth table? (Your answer: \_\_\_\_\_)
- (e) What is the sum of the above four earnings? (Your answer: \_\_\_\_\_)

1.2 Suppose that you are Member 2. What is **your** earning from

- (a) the first table? (Your answer: \_\_\_\_\_)
- (b) the second table? (Your answer: \_\_\_\_\_)

- (c) the third table? (Your answer: )
- (d) the forth table? (Your answer: )
- (e) What is the sum of the above four earnings? (Your answer: )
2. Suppose that you are Member 1 and Member 2's choice is  $(a_1, b_3, c_2, d_2)$ . What is your **highest possible** earning
- a. from the first table? (Your answer: )
- b. from the second table? (Your answer: )
- c. from the third table? (Your answer: )
- d. from the forth table? (Your answer: )
3. Suppose that you are Member 2 and Member 1's choice is  $(A_2, B_1, C_1, D_3)$ . What is your **highest possible** earning
- a. from the first table? (Your answer: )
- b. from the second table? (Your answer: )
- c. from the third table? (Your answer: )
- d. from the forth table? (Your answer: )

## Appendix D – Experimental Instructions for Treatment 2

### INSTRUCTION

You are about to participate in an experiment in the economics of decision-making. If you follow these instructions closely and consider your decisions carefully, you can earn a considerable amount of money, which will be paid to you in cash at the end of the experiment. To ensure best results for yourself, please **DO NOT COMMUNICATE** with the other participants at any point during the experiment. If you have any questions, or need assistance of any kind, raise your hand and one of the experimenters will approach you.

Today's experiments consist of **FOUR** sections. The final cash payment will be the sum of your earnings from the four sections, translated into HKD as the exchange rate of 1 token = 2 HKD, plus a show-up payment of HK\$40 for arriving to the experiment on time and participating. The following is the instruction for the first section. After you participate in the first section, further instructions will be given to you via your computer screen.

#### *Section 1*

You will play **eight** 4-player games. In each of these games, you will be randomly matched with other participants currently in this room. For each game, you will choose one of three actions. Each other participant in your game will also choose one of three actions.

<i>Your Earnings</i>					<i>Player 2's Earnings</i>					<i>Player 3's Earnings</i>					<i>Player 4's Earnings</i>				
2's action					3's action					4's action					Your action				
Your action		d	e	f	2's action		g	h	i	3's action		j	k	l	4's action		a	b	c
	a	10	4	16		d	12	16	4		g	20	12	8		j	10	12	8
	b	20	8	0		e	0	12	8		h	6	8	18		k	6	20	18
	c	4	18	12		f	4	4	20		i	0	16	4		l	16	4	0

Your earnings will depend on the combination of your action and player 2's action. These earnings possibilities will be represented in a table like the one above. Your action will determine the row of the table and player 2's action will determine the column of the table. You may choose action a, b, or c and player 2 will choose action d, e, or f. The cell corresponding to this combination of actions will determine your earnings. For example, in the above game, if you choose a and player 2 chooses d, you would earn 10 tokens. If instead player 2 chooses e, you would earn 4 tokens. Player 2, Player 3, and Player 4's earnings are listed in the other three tables. Player 2 may choose action d, e, or f, Player 3 may choose action g, h, or i, and Player 4 may choose action j, k, or l. Player 2's earnings depend upon the action he chooses and the action player 3 chooses. Player 3's earnings depend upon the action he chooses and the action Player 4 chooses. Player 4's earnings depend upon the action he chooses and the action you choose. For example, if you choose c, player 2 chooses e, player 3 chooses h, and player 4 chooses k, then you would earn 18 tokens, player 2 would earn 12 tokens, player 3 would earn 8 tokens, and player 4 would earn 18 tokens. The different earnings tables will appear in a random order for each game. As well, the earnings tables will differ from game to game. So you should always look at the earnings and order of the tables carefully at the beginning of each game.

<i>Player 2's Earnings</i>					<i>Player 4's Earnings</i>					<i>Your Earnings</i>					<i>Player 3's Earnings</i>				
3's action					Your action					2's action					4's action				
2's action		g	h	i	4's action		a	b	c	Your action		d	e	f	3's action		j	k	l
	d	12	16	4		j	10	12	8		a	10	4	16		g	20	12	8
	e	0	12	8		k	6	20	18		b	20	8	0		h	6	8	18
	f	4	4	20		l	16	4	0		c	4	18	12		i	0	16	4

When you start each new game, you will be randomly matched with different participants. We do our best to ensure that you and your counterparts remain anonymous. You will be required to spend at least 90 seconds on each game. You may spend more time on each game if you wish.

### *Earnings from Section 1*

For the payment from this section, *one* game will be randomly selected at the end of the

experiment. Every participant will be paid based on their actions and the actions of their randomly chosen group members in the selected game. Any of the games could be the game selected. So you should treat each game like it will be the one determining your payment. You will be informed of your payment, the game chosen for payment, what action you chose in that game, and the action of your randomly matched counterpart only at the end of the experiment. You will not learn any other information about the actions of other players in the experiment. The identity of your randomly chosen counterparts will never be revealed.

### Quiz

<i>Player 3's Earnings</i>					<i>Player 2's Earnings</i>					<i>Player 4's Earnings</i>					<i>Your Earnings</i>				
4's action					3's action					Your action					2's action				
3's action		j	k	l	2's action		g	h	i	4's action		a	b	c	Your action		d	e	f
	g	20	12	8		d	12	16	4		j	20	12	8		a	10	4	16
	h	6	8	18		e	0	12	8		k	6	8	18		b	20	8	0
	i	0	16	4		f	4	4	20		l	0	16	4		c	4	18	12

Consider the game above. Your earnings are given by the blue numbers. You may choose a or b or c.

- Your earnings depend on your action and the action of which other player?
  - Player 3
  - Player 2
  - Player 4
- Suppose you choose a, Player 2 chooses f, Player 3 chooses i, Player 4 chooses k. What will your earnings be?
  - 10
  - 0
  - 16
  - 6
- Suppose Player 2 chooses d, Player 3 chooses h, and Player 4 chooses j. Which action will give you the highest earning?
  - a
  - b
  - c



learn any other information about the actions of other players in the experiment. The identity of your randomly chosen counterpart will never be revealed.

### *Section 3 (presented only via the screen.)*

You will play **eight** 4-player games. In each of these games, you will be randomly matched with other participants currently in this room. For each game, you will choose one of three actions. Each other participant in your game will also choose one of three actions.

<i>Your Earnings</i>					<i>Player 2's Earnings</i>					<i>Player 3's Earnings</i>					<i>Player 4's Earnings</i>				
2's action					3's action					4's action					Your action				
Your action		d	e	f	2's action		g	h	i	3's action		j	k	l	4's action		a	b	c
	a	10	4	16		d	12	16	4		g	20	12	8		j	10	12	8
	b	20	8	0		e	0	12	8		h	6	8	18		k	6	20	18
	c	4	18	12		f	4	4	20		i	0	16	4		l	16	4	0

The rule of the games in this section is the same as the rule of the games in Section 1. However, the different earnings tables will appear in a random order for each game. As well, the earnings tables will differ from game to game. So you should always look at the earnings and order of the tables carefully at the beginning of each game.

When you start each new game, you will be randomly matched with different participants. We do our best to ensure that you and your counterparts remain anonymous. You will be required to spend at least 90 seconds on each game. You may spend more time on each game if you wish.

### *Earnings from Section 3*

For the payment from this section, *one* game will be randomly selected at the end of the experiment. Every participant will be paid based on their actions and the actions of their randomly chosen group members in the selected game. Any of the games could be the game selected. So you should treat each game like it will be the one determining your payment. You will be informed of your payment, the game chosen for payment, what action you chose in that game, and the action of your randomly matched counterpart only at the end of the experiment. You will not learn any other information about the actions of other players in the experiment. The identity of your randomly chosen counterparts will never be revealed.

### *Section 4 (presented only via the screen.)*

You will play **three** 2-player games. In each of these games, you will be randomly matched with another participant currently in this room. For each game, you will choose one of two/three actions. The other participant (player 2) in your game will also choose one of two/three actions. Your earnings will depend on the combination of your action and player 2's action. These earnings possibilities will be represented in a table like the one below. Your action will determine the row

of the table and the other player's action will determine the column of the table.

<i><b>Your Earnings</b></i>					
Your action	2's action				
		a	b	c	
	a	0	5	10	
	b	5	0	10	
	c	15	0	15	

<i><b>Player 2's earning</b></i>					
2's action	Your action				
		a	b	c	
	a	0	10	5	
	b	5	10	0	
	c	10	0	5	

When you start each new game, you will be randomly matched with different participants. We do our best to ensure that you and your counterparts remain anonymous. You will be required to spend at least 30 seconds on each game. You may spend more time on each game if you wish.

#### ***Earnings from Section 4***

For the payment from this section, *one* game will be randomly selected at the end of the experiment. Every participant will be paid based on their actions and the actions of the randomly chosen counterpart in the selected game. Any of the games could be the game selected. So you should treat each game like it will be the one determining your payment. You will be informed of your payment, the game chosen for payment, what action you chose in that game, and the action of your randomly matched counterpart only at the end of the experiment. You will not learn any other information about the actions of other players in the experiment. The identity of your randomly chosen counterpart will never be revealed.