Does Jump Bidding Increase Sellers' Revenue? Theory and Experiment*

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Abstract

We consider an environment in which bidders decide whether to jump bids in a simple two-bidder ascending auction with independent private values. We compare two types of equilibria, one that involves jump bidding and another that does not. We show that the revenue in the jump bidding equilibrium dominates that in the no-jump equilibrium when bidders are risk averse. Isolating the revenue impact of jump bidding from that of overbidding, our experimental design allows us to demonstrate that sellers' revenue increases due to jump bidding but only insignificantly so.

Keywords: English auction, jump bidding, risk aversion, laboratory experiment

JEL classification: C12; C90; D44; D82

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"A collective gasp swept the ballroom as the first round of results was announced: Bidding had started at $20 million each for two licenses and $10 million for five others. That was several times the sum that the experts had projected for the first round." The Washington Post, July 26, 1994, p. D1, on the Nationwide Narrowband PCS auction in July 1994

1 Introduction

A fundamental question in economics is how prices are formed in markets. Auctions provide an excellent framework to study this question. To sell an object in an auction, sellers choose the optimal auction rule to maximize profits, and buyers choose the optimal strategies to compete to win the object. That is, the price of the object sold is a result of the interaction between sellers and buyers.

In an ascending-price auction, the price gradually increases in fixed increments. During the auction, however, any bidder may call out a price that is much higher than the current price plus the increment. This "*jump-bidding is an endemic feature of real-world ascending auctions, including not only FCC wireless spectrum auctions but also online (eBay) auctions and conventional art and antiques auctions run by Sotheby's and Christie's for hundreds of years*" (Grether, Porter, and Shum [18]). For example, Cramton [11] documents that in an FCC radio spectrum auction, forty-nine percent of the new bids are jump bids.[1]

Why do bidders raise their prices voluntarily? An answer may be found in one of the most famous examples of jump bidding, as described below by Avery [2].

> *An infamous recent example occurred in 1988 when Ross Johnson, the CEO of RJR Nabisco, made a bid of $75 for the shares of his own company when the stock was trading at $55. In further competition. . . Kohlberg, Kravis and Roberts (KKR) raised Johnson's bid to $90. KKR won the bidding at a final price of $106 after only a few more rounds of bidding. Later, George Roberts admitted that his company would not*

[1] Avery [2], Cramton [10], [11], and Daniel and Hirshleifer [13] offer numerous real-life jump bidding examples. Isaac, Salmon and Zillante [23] present field data about jump bidding from US FCC spectrum license auctions and UK 3G spectrum auctions.

have competed if Johnson had started with a higher opening bid of $90 or more.

Researchers have had a fairly good understanding of jump bidding, albeit mostly on the buyers' side. By jump bidding to a high price, a bidder may have to bear the cost of paying more than necessary; however, the bidder may receive the benefit of driving his opponents to quit earlier in the bidding than they would have otherwise. When the latter force outweighs the former force, jump bidding occurs. Daniel and Hirshleifer [13] suggest that bidding is costly, and bidders jump bids and quit early to reduce costs[2]; Avery [2] suggests that the phenomenon known as the winner's curse drives jump bidding.[3]

The situation is, however, much less clear on the sellers' side. In particular, current theories (e.g., Avery [2], Banks, Olson Porter, Rassenti, and Smith [5], Cybernomics [9], Daniel and Hirshleifer [13], Hörner and Sahuguet [20]) suggest that jump bidding reduces sellers' revenue, a suggestion that is in sharp contrast to the fact that jump bidding continues to be allowed rather than forbidden by auctioneers worldwide. This issue therefore raises the following question:

Does jump bidding increase sellers' revenue?

If the answer to this question is no, then all auctioneers should forbid jump bidding as long as their main objectives are to maximize their revenue.[4] If the answer is yes, then something is clearly missing in our understanding of jump bidding. Thus, the objective of this paper is to investigate whether jump bidding increases sellers' revenue.

We consider a simple stylized model of a two-bidder ascending auction with independent private value (hereafter, IPV) in which bidders decide whether to jump bids. The auction proceeds with the jump stage, followed by the bidding stage. In the jump stage, each bidder simultaneously chooses the initial opening bid; in the bidding stage, a standard English auction with the starting price at the maximum of the initial bids is

[2]Daniel and Hirshleifer [13] consider an auction with sequential bids and construct an equilibrium in which the first bidder uses a monotonic bidding strategy that fully reveals his/her value to the second bidder. After observing a bid from the first bidder, the second bidder may understand that there is no chance to win and thus want to quit early to reduce the bidding costs.

[3]By jump bidding, a bidder signals his intent to follow a more aggressive strategy. As a result, his competitors choose a less aggressive strategy and quit early because of the winner's curse.

[4]Alternatively, one would need to develop a new theory of optimal auction design without assuming revenue maximization. In any case, providing an answer to the question is the first important step.

conducted, and each bidder chooses the price at which to exit. We find that this game has two types of equilibria, one in which jump bidding takes place to signal one's value and another in which no such jump bidding occurs. Moreover, we show that these two types of equilibria are Pareto ranked when bidders are risk averse. The jump bidding equilibria dominate no-jump equilibria in revenue.

To illustrate the main idea, consider a simple example of a 2-bidder English auction with the uniform prior. Consider the possibility that jumping from the current price of \$0 to \$700 can signal that one's value is greater than or equal to \$1,000. Suppose that my value of an object for sale is \$1,000. By viewing such a signal, all types of my opponent with values of less than $\$1,000$ would quit immediately because they expect to have no chance to win. By paying a price of \$700, I receive a benefit by deterring my competitor with values up to $1,000$; therefore, I prefer jump bidding. To make the above strategy profile an equilibrium, the signaling must be credible (i.e., I prefer not to jump bid if my value is less than $\$1,000$), which can be supported by a certain type of risk attitudes. For example, the non-decreasing absolute risk aversion introduced in McAfee and Vincent [31] is sufficient to support the equilibrium described above. Furthermore, suppose that I am risk averse. In that case, jump bidding serves as insurance: for both jumping and not jumping, I win over the same group of types of my opponent (i.e., types with value less than $\$1,000$), but I pay a fixed price of \$700 for jumping rather than a random winning price between \$0 and $\$1,000$ (with an expected value of \$500) for not jumping. Hence, by jump bidding, I take the insurance and surrender some risk premium ($\$200 = \$700 - \$500$) to the seller. As a result, the seller's revenue increases.

To the best of our knowledge, little empirical and experimental research has been undertaken on the revenue implications of jump bidding, and we design lab experiments to answer this question.[5] Compared to other methodologies (e.g., empirical analysis, field experiments), lab experiments are able to fully control all other economic factors except

[5]Isaac, Salmon, and Zillante [22] are the first researchers to use laboratory experiments to test various jump bidding models. Their study, which focuses on jump bidding on the bidders' side, indicates that the jump bidding observed in field auctions is likely linked to bidders' impatience. Recently, Grether, Porter, and Shum [18] adopted the field experiment approach and manipulated the price grid, the possible amounts that bidders can bid above the current price, on online auction sites that sell used automobiles via ascending auctions. These researchers find peculiar patterns of bidding, suggesting that bidders are "cyber-shills" working on behalf of sellers.

jump bidding and to distill the pure revenue effect of jump bidding.[6]

We implement a simple two-bidder ascending auction game in the laboratory via the clock auction implementation *à la* Kagel, Harstad and Levin [26]. Three treatment conditions were considered; the first two treatments had an initial stage in which each bidder could choose to jump bid or not. The only difference across the two treatments is whether any jump amount is allowed (in the *Baseline* treatment) or only a particular jump amount (in the *Binary* treatment) is allowed. The third treatment is the *No-jump* treatment in which jump bidding is not allowed. The data from the three treatments show that the revenues obtained in the two treatments that allow jump bidding are significantly and substantially higher than the hypothetical revenue calculated based on the dominant-strategy equilibrium. However, the revenue obtained in the *No-jump* treatment is also substantially higher than the hypothetical revenue and not statistically different from the revenues obtained from the *Baseline* and *Binary* treatments. This observation suggests that a substantial degree of *overbidding* exists in our experiment, a well-documented phenomenon in ascending auction experiments, i.e., bidders remain in an auction even if the prices exceed their values (see, e.g., Kagel, Harstad, and Levin [26], Kagel and Levin [25]).[7] The degree of overbidding observed in the *No-jump* treatment is significantly more substantial than those in the other two treatments.

To understand the revenue implications of jump bidding, we thus have to eliminate overbidding.[8] We attempt to eliminate overbidding by designing an *Amended Random Payment* (*ARP*) scheme: 10 rounds of English auctions are conducted, and only one round is randomly and independently chosen to be the payment round for each bidder, i.e., each bidder's final payoff depends only on the outcome of the auction in his/her pay-

[6]Another potential difficulty in other methodologies is how to estimate the value distribution from the jump prices, which is necessary to determine the revenue effect of jump bidding.

[7]However, as shown in several papers (e.g., Kagel, Harstad, and Levin [26], Garratt, Walker and Wooders [16]), overbidding is common only among inexperienced bidders and tends to be a short-term phenomenon. Malmendier and Lee [28] empirically identify overbidding in eBay auctions by comparing online auction prices to fixed prices for the same item on the same website. Although only a small fraction of bidders are identified as overbidders, these bidders generate a large fraction of auctions with overbidding. The results are explained by limited attention.

[8]Section 4 presents data from three treatments with no experimental control for overbidding and shows that actual revenues from the treatments with and without jump bidding are not significantly different. Our exit survey results reveal that overbidding is primarily induced by spitefulness, which is consistent with the findings in the literature (Andreoni, Che and Kim [1], Cooper and Fang [8]).

ment round.[9] Furthermore, none of the bidders know their payment round, and hence, in every round, every bidder attempts to maximize profits. However, we amended the standard random payment scheme by allowing every bidder to *know* whether each round is a payment round for his/her opponent. As a result, in every nonpayment round for the opponent, any concern related to other-regarding preferences is fully eliminated for every bidder. Our experimental data show that our *ARP* design eliminates 76% of overbidding. Successfully isolating the overbidding phenomenon, the data from the *ARP* design clearly demonstrate the revenue implications of jump bidding. The revenue obtained from the treatment with jump bidding is significantly and substantially higher than that obtained from the treatment without jump bidding.

The remainder of the paper proceeds as follows. In Section 2, we present a model of jump bidding with risk aversion. In Sections 3 and 4, we discuss the experimental design and findings, respectively. Section 5 presents our new experimental design to control for overbidding behavior. Section 6 provides regression results on the determinants of jump bidding. Section 7 concludes the paper. A review of the related literature is presented in the remainder of this section.

1.1 Related Literature

This section is devoted to a review of the related literature. In particular, we discuss several theoretical and experimental papers that attempt to explain why buyers participate in jump bidding.

Given private values, it is well known that bidding one's true value is a weakly dominant strategy in a 2nd-price auction, which is strategically equivalently to the classical English auction without jumps. This leads to the following question: why do bidders jump bid? Two papers have provided well-reasoned answers. Daniel and Hirshleifer [13] argue that bidding is costly and that bidders jump bid to reduce costs. In their model, bidders are risk neutral, and thus revenue equivalence holds without bidding costs. With

[9]This"random lottery incentive system" is widely used to motivate subjects in an experiment with multiple rounds. Cubitt, Starmer and Sugden [12] provide evidence for the validity of the incentive system. Azrieli, Chambers and Healy [3] show that under a mild assumption imposed on subjects' preferences, the random lottery incentive system is the only incentive-compatible mechanism.

bidding costs, revenue decreases to compensate for the effect induced by the bidding costs. Avery [2] studies an affiliated-value model and argues that the winner's curse drives jump bidding. By jump bidding, a bidder signals his/her intent to follow a more aggressive strategy. As a result, his/her competitors choose a less aggressive strategy and quit early to avoid the winner's curse.

Since Avery [2] and Daniel and Hirshleifer [13], several other papers have investigated jump bidding in various contexts. Easley and Tenorio [14] study internet auctions and explain jump bidding by entry costs and uncertainty over future entries. Gunderson and Wang [19], Hörner and Sahuguet [20] and Zheng [36] also construct jump-bidding equilibria in private-value models. Gunderson and Wang [19] assume a disconnected support of bidders' values and suggest that jump bidding is a signal of high value. Instead of focusing on why people engage in jump bidding, Hörner and Sahuguet [20] examine how *bluffing* and *sandbagging* (i.e., nonmonotone bidding strategies) are implemented in jump bidding. Zheng [36] considers multiunit auctions and studies jump bidding as a signaling device across auctions. In all of these papers, jump bidding is regarded as a signal by the bidder. The difference among them is what makes the signaling credible. We relate jump bidding to risk attitudes and offer a novel explanation. Goeree [17] studies a modified English auction followed by aftermarket competition.[10] Bidders signal high values in the English auction to gain an advantage in the aftermarket, and sellers' revenue increases as a result. Signaling occurs only when all bidders but one have dropped out of the auction, with the final bidder remaining in the auction until the price reaches the optimal signaling value. However, under the traditional English auction rules that we consider, such signaling is excluded because the auction ends immediately when only one bidder remains.

Other authors explain jump bidding using behavioral reasons. Bidders' irrationality may be a reason for jump bidding, as shown in Rothkopf and Harstad [34]. Although Isaac, Salmon, and Zillante [23] model jump bidding as a strategic dynamic game, information updating in their game is non-strategic.[11] Malmendier and Lee [28] propose that

[10] The main objective of Goeree [17] is to compare signaling effects in first-price, second-price and English auctions when aftermarket competition exists.

[11] In the model of Isaac et al. [23] (page 150), after seeing a jump bid of p from bidder $-i$, bidder i forms a naive belief of $[v_{-i} \geqslant p]$, although only bidders $-i$ with $v_{-i} \geqslant \overline{v} > p$ make such a jump in equilibrium, i.e., bidders do not use information from Bayesian updating according to the equilibrium strategy profile

people may bid above their true values for behavioral reasons: limited memory, limited attention, joy of winning, etc. Isaac et al. [22] use experiments to test various jump-bidding models, finding that the jump bidding observed in field auctions is likely the result of bidders' impatience.

The idea that sellers can provide some sort of insurance to risk-averse buyers to generate more revenue was discussed in a few earlier papers. In particular, Budish and Takeyama [6] and Reynolds and Wooders [33] both suggest that a buy-it-now price can serve as insurance for risk-averse bidders and thus increases sellers' revenue. In their models, the seller also offers the buyer an opportunity to pay a (potentially) higher fixed price to avoid uncertainty. The buyer takes the fixed price and surrenders some risk premium to the seller. As a result, the seller's revenue increases. In both Budish and Takeyama [6] and Reynolds and Wooders [33], however, the buy-it-now option is chosen exogenously by the seller. In contrast, jump bidding in our paper is chosen endogenously by the buyers and thus plays a signaling role.

Auctions with risk-averse bidders have been explored previously. For example, Maskin and Riely [29] and Matthews [1] study the impact of risk aversion on revenue, with the former taking the seller's view and the latter taking the buyer's view. More recently, Smith and Levin [35] and Li, Lu, and Zhao [27] study risk-averse bidders with endogenous entry. Hu, Matthews, and Zou [21] study the impact of risk aversion on the optimal reserve price. Bajari and Hortaçsu [4] show that a model assuming risk-averse bidders can explain their experimental auction data better than an alternative model with risk-neutral bidders. To the best of our knowledge, we are the first to associate risk aversion with jump bidding.

2 A Theory of Jump Bidding

We consider a 2-bidder IPV setup. We model the auction by a 2-stage game: the jump stage (Stage 1), followed by the bidding stage (Stage 2). In Stage 1, each bidder simultaneously chooses the initial bid (i.e., the jump bid); in Stage 2, a standard English auction

and all prior information.

with the starting price at the maximum of the initial bids is conducted, and each bidder chooses the price at which to exit. This simple stylized model à la Avery [2] is a workhorse for us to demonstrate the revenue implications of jump bidding, and it is the simplest possible way to understand the role of jump bidding in the seller's revenue.[12]

One indivisible object is for sale, with bidders 1 and 2 having values v_1, v_2, respectively. The values have an i.i.d. distribution on the support $[0,1]$ with cdf $F(\cdot)$. Let bidder $-i$ denote bidder i's opponent. The two bidders are expected utility maximizers with the same differentiable and strictly increasing Bernoulli utility function $u(\cdot)$. We normalize $u(0)$ to 0. Suppose that $u'''(t)$ exists. We list a few assumptions on risk attitudes as follows.

$$\text{risk-neutral bidders: } u''(t) = 0; \quad \text{risk-averse bidders: } u''(t) < 0,$$
$$\text{CARA bidders: } \frac{d\left(-\frac{u''(t)}{u'(t)}\right)}{dt} = 0; \qquad \text{IARA bidders: } \frac{d\left(-\frac{u''(t)}{u'(t)}\right)}{dt} > 0.$$

In Stage 1, each bidder i chooses a jump bid $\beta_i \in [0,1]$; in Stage 2, an English auction with starting price $\max\{\beta_i, \beta_{-i}\}$ is conducted, and each bidder i chooses the price $b_i(\beta_i, \beta_{-i}, v_i) \geqslant \max\{\beta_i, \beta_{-i}\}$ at which to exit. Bidder i wins the auction if $b_i(\beta_i, \beta_{-i}, v_i) > b_{-i}(\beta_i, \beta_{-i}, v_{-i})$. A tie occurs if $b_i(\beta_i, \beta_{-i}, v_i) = b_{-i}(\beta_i, \beta_{-i}, v_{-i})$. When a tie occurs, bidder i wins if $b_i(\beta_i, \beta_{-i}, v_i) = b_{-i}(\beta_i, \beta_{-i}, v_{-i}) = \beta_i > \beta_{-i}$, and in all other cases, a fair coin determines the winner. Suppose that bidder i is the winner. Then, he/she wins at price $b_{-i}(\beta_i, \beta_{-i}, v_{-i})$ and obtains utility $u[v_i - b_{-i}(\beta_i, \beta_{-i}, v_{-i})]$, while bidder $-i$ gets 0.

Throughout the paper, we adopt the solution concept of perfect Bayesian equilibrium (PBE). As a benchmark, the usual no-jump equilibrium is defined as follows.

$$\widehat{\sigma}_i : \left(\begin{array}{c} \text{stage 1: } \beta_i = 0; \\ \text{stage 2: } b_i(\beta_i, \beta_{-i}, v_i) = \max\{\beta_i, \beta_{-i}, v_i\}. \end{array} \right)$$

Following $(\widehat{\sigma}_1, \widehat{\sigma}_2)$, no one jumps in Stage 1, and each bidder remains in the auction in Stage 2 until the price reaches his/her true value. The usual argument shows that $(\widehat{\sigma}_1, \widehat{\sigma}_2)$

[12] Clearly, real-life jump bidding is much more complicated. For instance, it may involve multiple bidders and multiple rounds of jump bidding. However, it is logically straightforward but tedious to extend our model to a setup with multiple bidders and multiple rounds of jump bidding, and all of our results could be extended to such a setup with the intuition remaining the same. See Appendix E for further discussion of equilibria with two rounds of jump bidding and its revenue implications.

is a PBE.

Proposition 1 (no-jump PBE). *$(\widehat{\sigma}_1, \widehat{\sigma}_2)$ is a PBE.*

We now define a class of jump bidding equilibria. For any $v \in [0,1]$, define $k(v) \in [0,1]$ to be the unique number satisfying[13]

$$F(v)u(v - k(v)) = \int_0^v u(v - v')dF(v'). \tag{1}$$

Fix any $(v^*, k(v^*)) \in (0,1) \times (0,1)$; we construct a jump bidding equilibrium (σ_1^*, σ_2^*) as follows.[14]

$$\sigma_i^* : \left(\begin{array}{c} \text{stage 1: } \beta_i = \begin{cases} 0, & \text{if } v_i < v^*; \\ k(v^*), & \text{if } v_i \geqslant v^*. \end{cases} \\ \text{stage 2: } b_i\left(\beta_i, \beta_{-i}, v_i\right) = \begin{cases} \max\left\{\beta_i, \beta_{-i}\right\}, & \text{if } \beta_{-i} = k(v^*) \text{ and } v_i < v^*; \\ \max\left\{\beta_i, \beta_{-i}, v_i\right\}, & \text{otherwise.} \end{cases} \end{array} \right)$$

By following σ_i^*, the high types of bidder i (i.e., $v_i \geqslant v^*$) jump to $k(v^*)$ in Stage 1 and the low types of bidder i (i.e., $v_i < v^*$) do not jump. That is, bidder i uses the jump bid $k(v^*)$ to signal his/her high values. In the case in which $\beta_{-i} = k(v^*)$ and $v_i < v^*$, bidder i infers that $v_{-i} \geqslant v^*$ and expects no chance to win, and hence, bidder i finds it a best reply to quit immediately. In any other case, bidder i follows the weakly dominant strategy $b_i\left(\beta_i, \beta_{-i}, v_i\right) = \max\left\{\beta_i, \beta_{-i}, v_i\right\}$ in the clock auction in Stage 2.

Proposition 2. *(σ_1^*, σ_2^*) is a PBE for risk-neutral, CARA and IARA bidders.*

We provide an intuition of the proof here. To make (σ_1^*, σ_2^*) a PBE, the jump bid must be credible, i.e., the high types prefer "jumping to $k(v^*)$" to "no jump," and the low types prefer "no jump" to "jumping to $k(v^*)$. First, consider the threshold type v^*, and for both options, he/she wins the auction if and only if $v_{-i} \leqslant v^*$. In addition, the only difference

[13] $F(v)u(v-y)$ is strictly decreasing in y. Because $F(v)u(v-0) \geqslant \int_0^v u(v-v')dF(v') \geqslant F(v)u(v-v)$, there exists a unique $k(v)$ for each $v \in [0,1]$ such that equation (20) is satisfied.

[14] Rigorously, we need to specify beliefs on off-equilibrium paths for a PBE. For notational ease, we omit it because off-equilibrium paths occur only in Stage 2 (i.e., a standard English auction), and bidding the true value is a weakly dominant strategy regardless of beliefs. As a result, our equilibrium survives the equilibrium refinement that imposes an additional restriction on off-path beliefs (e.g., the intuitive criteria).

is that she wins at the fixed price $k(v^*)$ for "jumping to $k(v^*)$," while he/she wins at a random price $v_{-i} \sim [0, v^*]$ with cdf $\frac{F(v_{-i})}{F(v^*)}$ for "no jump." By (20), $k(v^*)$ is defined to make type v^* indifferent between the two options. Second, consider a high-type bidder i (i.e., $v_i > v^*$). He/she faces exactly the same dilemma as type v^*: Conditional on $v_{-i} \leqslant v^*$, he/she wins at the fixed price $k(v^*)$ for "jumping to $k(v^*)$," while he/she wins at a random price $v_{-i} \sim [0, v^*]$ with cdf $\frac{F(v_{-i})}{F(v^*)}$ for "no jump. "[15] Given CARA/IARA, this high type is weakly/strictly more risk averse than type v^*; i.e., he/she is willing to sacrifice weakly/strictly more to eliminate the same risk. Hence, this high type weakly/strictly prefers "jumping to $k(v^*)$" (i.e., the fixed price $k(v^*)$) to "no jump" (i.e., the random price $v_{-i} \sim [0, v^*]$ with cdf $\frac{F(v_{-i})}{F(v^*)}$). Similarly, a CARA/IARA low type (i.e., $v_i < v^*$) weakly/strictly prefers "no jump" to "jumping to $k(v^*)$."

The following theorem explains why jump bidding increases revenue.

Theorem 1 (Seller's Revenue)**.** *Given risk-averse bidders, the strategy profile* (σ_1^*, σ_2^*) *leads to a higher expected revenue for the seller than* $(\widehat{\sigma}_1, \widehat{\sigma}_2)$*.*

Theorem 1 compares two strategy profiles (σ_1^*, σ_2^*) and $(\widehat{\sigma}_1, \widehat{\sigma}_2)$ that may not necessarily be an equilibrium. In Proposition 2, we show that risk-neutral, CARA and IARA are *sufficient* conditions for (σ_1^*, σ_2^*) to be an equilibrium. However, they are not necessary conditions. Moreover, there may be cases in which risk aversion holds and CARA or IARA fail; nevertheless, a jump bidding equilibrium exists. Theorem 1 covers such cases and compares the seller's revenue from the two different types of strategy profiles.

The intuition for Theorem 1 is straightforward: jump bidding serves as insurance. By jump bidding, high types (of bidder i) pay the fixed price $k(v^*)$ to insure themselves against random winning price $v_{-i} (\leqslant v^*)$. If the bidders are risk averse, they surrender the risk premium to the seller.

Furthermore, both bidders are weakly better off in the jump bidding equilibrium. Let $Eu_i(v_i | \varepsilon_i, \varepsilon_{-i})$ denote the expected utility of bidder i with value v_i when the strategy profile $(\varepsilon_i, \varepsilon_{-i})$ is chosen by the two bidders.

[15] Conditional on $v_{-i} > v^*$, the two options induce the same outcome.

Theorem 2 (Bidders' Welfare). *For a PBE* (σ_1^*, σ_2^*), $Eu_i\left(v_i|\sigma_i^*, \sigma_{-i}^*\right) \geqslant Eu_i\left(v_i|\widehat{\sigma}_i, \widehat{\sigma}_{-i}\right)$ *for every* $i \in \{1,2\}$ *and every* $v_i \in [0,1]$.

Theorems 1 and 2 suggest that the jump-bidding equilibrium (σ_1^*, σ_2^*) is (ex ante and interim) weakly Pareto superior to the no-jump equilibrium $(\widehat{\sigma}_1, \widehat{\sigma}_2)$. The proofs of Proposition 2 and Theorems 1 and 2 can be found in Appendix B.

Note that our focus is on understanding the revenue effect qualitatively rather than identifying specific equilibria or measuring the exact revenue impact quantitatively. Clearly, our construction of one-threshold jump-bidding equilibria can be extended in a straightforward way to multithreshold equilibria and/or equilibria in which bidders have different thresholds for jumps. Nevertheless, all these equilibria share the same qualitative revenue effect, i.e., jump bids serve as insurance, and the seller's revenue increases when risk-averse bidders follow a jump-bid equilibrium. For example, Appendix E presents the construction of two-threshold jump-bidding equilibria and shows that it qualitatively generates the same revenue implications.

3 Experimental Design

In our experimental design, there were a total of three treatments, as summarized in Table 1.

Table 1: Experimental Treatments

	Baseline	*Binary*	*No-Jump*
Jump bids allowed in Stage 1	Any integer in $[0, 60]$	0 or 20	Not Allowed

We used the uniform value distribution over the support $\{0, 1, ..., 60\}$ for our experimental implementation. The three treatments differ only with respect to what was allowed in Stage 1. In the *Baseline* treatment, individuals are allowed to make an initial bid of any integer in $[0, 60]$ inclusively. In the *Binary* treatment, the initial bid is a binary choice between 0 and 20. In the *No-Jump* treatment, the initial jump bid is not allowed, i.e., the initial bid must be 0.

There are two reasons to include the *Binary* treatment in our design, although having the binary options for the initial bid may not appear very natural. First, it is standard in the literature (e.g., Avery [2]) to model jump bidding by having an initial bid stage with only binary options. Second, the natural setup with more than two options in the initial-bid stage as in our *Baseline* treatment may suffer from the problem of multiple equilibria because we are free to choose the out-of-equilibrium belief for any initial bid never made in equilibrium. By offering only the binary options, we attempt to alleviate the problem of multiple equilibria and make a particular initial bid more focal.[16] The choice of 20 for the jump bidding allowed in the *Binary* treatment is guided by two considerations: 1) the jump amount is not too high relative to the upper bound of the value distribution such that we expect to observe jump bidding reasonably frequently, and 2) it is not too low such that the revenue impact is substantial in magnitude if jump bidding occurs.

3.1 Experimental Procedure

The experiment was conducted in English using z-Tree (Fischbacher, [15]) at the Hong Kong University of Science and Technology Experimental Laboratory. Two sessions each for the *Baseline* and *Binary* treatments and three sessions for the *No-jump* treatment were conducted using a between-subjects design. Each session involved *two independent matching groups*, each of which had five pairs of individuals. In total, 140 subjects participated in 7 sessions.[17] Subjects had no prior experience in our experiments and were recruited from the undergraduate/graduate population of the university.

Upon arrival at the lab, subjects were instructed to sit at separate computer terminals. Each was given a copy of the experimental instructions (see Appendix C). The

[16]More formally, one can show that there is a unique jump-bidding equilibrium in the environment with the binary initial bids $\{0, k\}$ with $k \in (0, 1)$ for risk-neutral, CARA and IARA bidders. The unique jump-bidding equilibrium is σ_i^* presented in Proposition 2 with $\beta_i = k$ if and only if $v_i \geqslant v^*$ where $v^* \in (0, 1)$ is uniquely identified.

[17]In Section 5, we present data from two additional treatments, each of which has two sessions. When including these four sessions with 80 additional subjects, we had 220 subjects participating in 11 sessions.

instructions were read aloud and supplemented by slide illustrations. In each session, subjects first participated in one practice round and then 10 official rounds. At the beginning of each session, participants were divided into two equal-sized matching groups. Within the matching group, participants were randomly matched to form a two-person pair in each round and randomly rematched after each round to form new pairs. Thus, each matching group provides an independent observation for our subsequent data analysis.

We illustrate the instructions for the *Baseline* treatment. The full instructions can be found in Appendix C. At the beginning of each round, the computer randomly drew a value for each individual with equal probability in the range between 0 and 60. Prior to the first session of our experiments, we randomly and independently drew the set of values for each individual and for each round and used it for all sessions (and all matching groups) to have a tight revenue comparison across treatments.[18] Each subject was privately informed about his/her own value but not those of others. In each round, each subject was endowed with 60 tokens and was asked to make a bid to win an auction that consisted of the following two stages: Initial Bidding Stage (Stage 1) and Price Clock Stage (Stage 2).[19]

In the initial bidding stage, subjects were asked to place an initial bid of any integer number between 0 and 60 inclusively.[20] The maximum of the initial bids in a pair became the initial price in the second stage. After all subjects submitted their initial bids, the initial price was announced for each pair, and they were asked to stay with the screen for a number of seconds – randomly determined between 5 seconds and 15 seconds – to consider what to do in the next stage. The waiting time was independent of the initial bids. If one's submitted initial bid was strictly lower than his/her opponent's initial bid, he/she was asked to decide whether to continue or to opt out. If one opted out, his/her opponent won the auction with the initial price. Otherwise, we proceeded to Stage 2. If a bidder submitted an initial bid higher than or equal to his/her opponent's initial bid,

[18]Table 11 presented in Appendix A reports the values used in the experiment.

[19]Using an ascending clock procedure whereby the price of an item increases in small fixed increments has been a standard way to implement an English auction in the laboratory since Kagel, Harstad and Levin [26].

[20]In the *Binary* treatment, however, only two options, 0 and 20, were given for the initial bid. No such stage existed in the *No-jump* treatment.

he/she was asked to click the continue button to proceed to Stage 2.

In the second stage, a price clock was presented on every bidder's decision screen that had three pieces of information: the (1) initial price, (2) current price, and (3) bidder's private value. The price clock started with the initial price determined in the initial bidding stage. The current price was displayed at the center of the clock, and every two seconds, the current price increased by 1 unit. The bidder's private value is highlighted in the clock in blue. A button labeled "Not Interested Anymore" was located under the price clock. When one of the individuals in each pair clicked the button, the price clock stopped, and the auction ended. The individual who remained in the auction was declared the winner and won the object at the price displayed on the clock. If no one dropped out until the current price was equal to 60, the auction ended, and the final price became 60. In this case, each individual had an equal chance of winning the auction. If one did not win the auction, the earnings became the endowment, i.e., 60, and otherwise, the earnings became the endowment plus the value minus the final price. At the end of each round, information feedback was provided, such as one's value, his/her opponent's value, the initial bid, his/her opponent's initial bid, the final price, the auction outcome and the final earnings.

At the end of each session, we elicited the risk attitude of each individual using two rounds of decision tasks according to Table 12 in Appendix A. Note that when subjects made decisions in the main auction game, we did not inform them that they would have additional tasks. Each decision task involved twelve rows, each presenting a decision problem that asks to choose between a simple lottery (Option A) and a certain outcome $\$Y$ (Option B). The simple lottery had two possible outcomes $\$(Y + h)$ and $\$(Y - h)$, and the probability assigned to the larger payoff monotonically increased as the row number increased. The decision problem in Row 4 in each task had a certain outcome that was exactly the certainty equivalent of the uncertain outcome such that a risk-neutral agent should be indifferent between the two options. The two rounds of decision tasks differed in terms of stake size: $Y = 10$ in the first task and $Y = 30$ in the second task, while $h = 4$ in both tasks.

We first identify individuals' risk attitudes based on the switching points in the two decision tasks. We classify a subject as risk averse (loving) if his/her switching points in both tasks were weakly above (below) Row 4 with at least one of them strictly above (below) Row 4. If the switching points in both tasks were exactly Row 4, we classify the subject as risk neutral. A total of 81.4% of subjects' risk attitudes are identified, while the rest (18.6%) are unidentified due to multiple switching points, at least in task, reverse switching (switching from Option A to Option B), or inconsistent risk attitudes across two rounds. Among the identified subjects, approximately 79% are classified as risk averse, while risk neutral and loving each represent slightly above 10%.

We further classify a subject as CARA, IARA, and DARA if his/her switching point in the task with a larger stake is the same, strictly higher, and strictly lower than the switching point in the task with a smaller stake; 86.4% of subjects' absolute risk aversions are identified.[21] Among the identified subjects, slightly more than 50% are classified as CARA, and approximately 28% are classified as IARA. Figure 7 and Table 13 in Appendix A report elicited risk attitudes at the aggregate level and the individual level, respectively.

We randomly selected one round of auctions for the real payment. A subject was paid the amount of tokens (1 token = 1 HKD) he or she earned in the selected round of the auction game and from the tasks for the risk attitude elicitation plus an HKD 30 show-up fee. Subjects earned on average HKD 140 ($\approx$ USD 18); the range was from HKD 126 to HKD 175.[22]

4 Experimental Findings

4.1 Bidding Behavior

We begin this section by drawing the reader's attention to Figures 1 and 2 below, which present subjects' bidding behavior in the *Baseline* and *Binary* treatments, respectively. Both figures contain four panels, with each panel referencing a different matching group.

[21]We can identify an individual's absolute risk aversion as long as a subject has a single switching point in each round regardless of whether the risk attitudes are consistent across the two rounds. We thus have a higher percentage of successful identification (86.4%) for absolute risk aversion than for risk attitude (81.4%).

[22]Under the Hong Kong currency board system, the HK dollar is pegged to the US dollar at the rate of 1 USD = 7.8 HKD.

For the sake of presentational convenience, we created the notion of *pairs* for each matching group as a collection of two-person pairs, one from each round, that were assigned to have particular (predetermined) realizations of values. For example, Pair 1 consisted of a pair with realized values 5 and 25 in Round 1, 44 and 12 in Round 2, ..., 41 and 30 in Round 10, and was so in every matching group in every session and treatment. Thus, two individuals in a given pair were not always the same across rounds.[23] The horizontal axis of each panel consists of five blocks, each of which refers to one pair. On each block is the period ranging from 1 to 10. On the vertical axis are three different prices for a given pair in a given period. The three prices shown in the figures are the initial price (IP, represented by the dark bar), final price (FP, represented by the thin dashed line), and hypothetical price (HP, represented by the solid line), where the initial price is the maximum of the two initial bids in each pair, the final price is the price at which the price clock stops, and the hypothetical price is the second-highest value in each pair.[24] Note that the solid hypothetical price lines in all panels in both figures look the same because we used a common value distribution and presented them according to the same order based on the notion of pairs for all sessions and treatments.

Four features of the data clearly emerge from both treatments. First, jump bidding is prevalent: 85.5% of the observations in the *Baseline* treatment and 28% of the observations in the *Binary* treatment had a strictly positive initial price.[25] Second, in most cases – 72% in the *Baseline* treatment and 66% in the *Binary* treatment – the final price is in the neighborhood (± 1) of the hypothetical price. Third, there are a few instances in which *overbidding* is observed; i.e., the final price is strictly higher than the hypothetical price. Fourth, there are a few instances in which *underbidding* is observed; i.e., the final price is strictly lower than the hypothetical price. Table 2 provides a summary of the bidding behavior in all three treatments.

When reporting our data, we distinguish three scenarios for both overbidding and

[23]By doing so, we can always keep the order of the presentation of results constant across all different matching groups, sessions, and treatments based on the predetermined realized values.

[24]We call the second-highest value in each pair the *hypothetical price* because if everyone hypothetically follows the weakly dominant strategy of quitting when the price reaches one's value, then the final price should be the second-highest value in the pair.

[25]Focusing on jump bidding significantly larger than 0, we report that 32% of subjects in the *Baseline* treatment made an initial bid weakly greater than 5.

Figure 1: Bidding Behaviors – *Baseline* Treatment

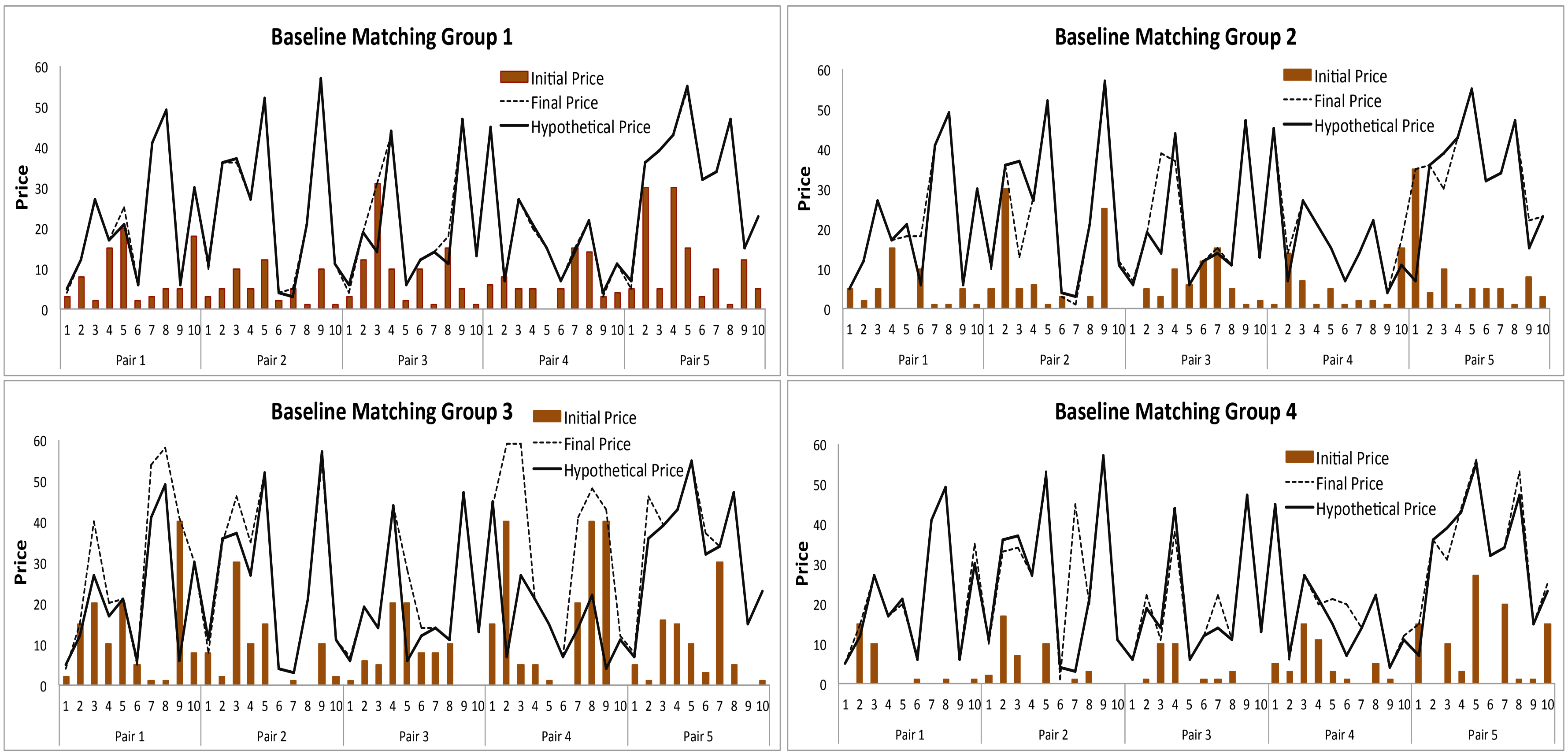
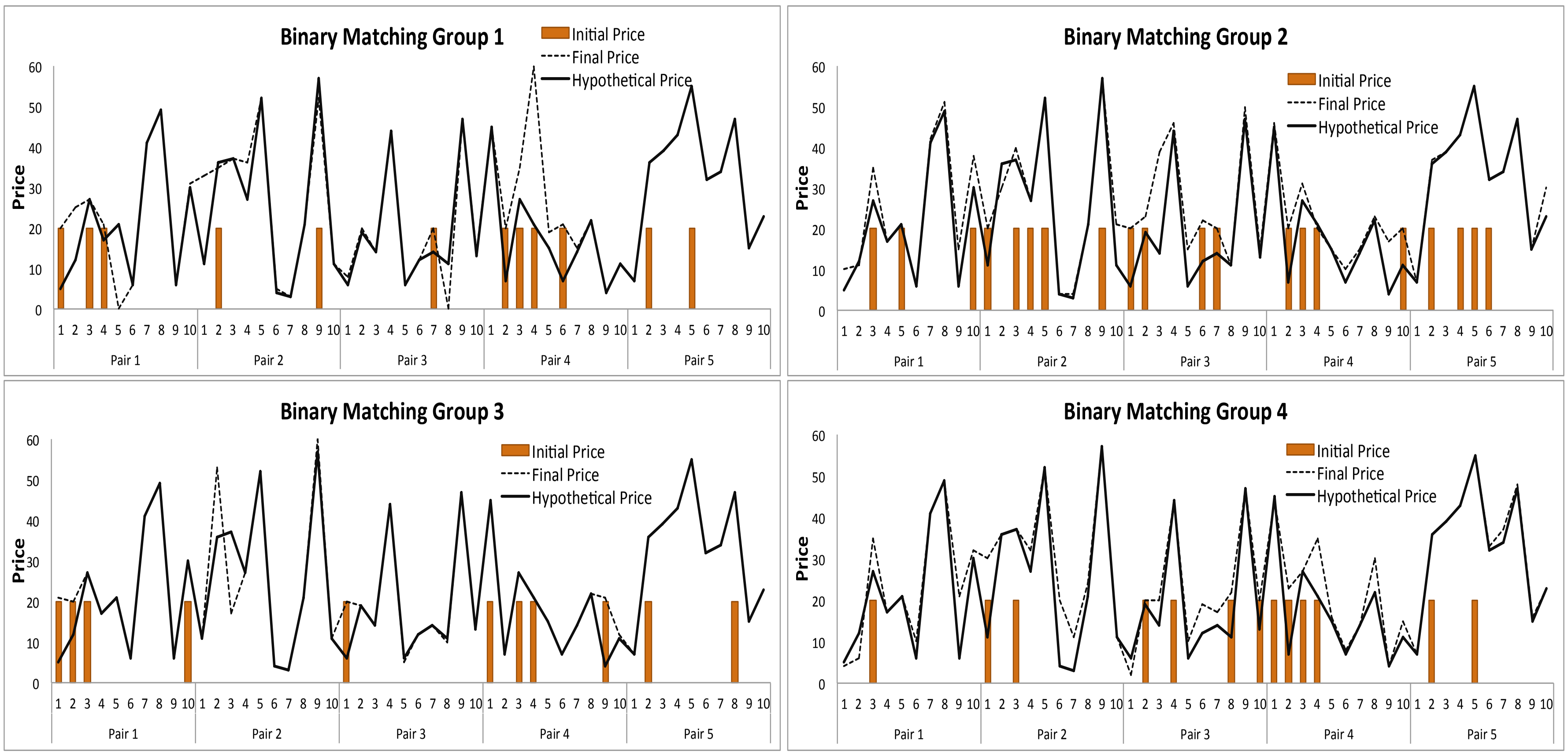


Figure 2: Bidding Behaviors – *Binary* Treatment



underbidding and categorize them into 6 subgroups as follows. First, we say that overbidding (underbidding) is observed if the final price is strictly higher (lower) than the hypothetical price $+1$ (-1).[26] Second, we say that overbidding (underbidding) is driven directly by jump bidding if one bidder jumps and the other quit immediately, indirectly driven by jump bidding if one bidder jumps and the other does not quit immediately, and not driven by jump bidding if no one jumps. Note that overbidding driven directly by jump bidding is consistent with the behavioral predictions of our theory.

Table 2: Bidding Frequencies

		Baseline					***Binary***					***No-jump***						
Matching Group		1	2	3	4	Mean	1	2	3	4	Mean	1	2	3	4	5	6	Mean
FP= HP (± 1)		44	39	31	34	37	35	26	43	28	33	24	33	24	36	26	34	29.5
OB	JB Direct	2	2	2	2	2	5	5	4	1	3.75	N/A						
	JB Indirect	2	3	10	1	4	2	6	0	5	3.25							
	No-jump	0	1	5	7	3.25	5	12	2	14	8.25	21	12	25	13	24	16	18.5
UB	JB Direct	2	1	1	2	2.5	2	0	0	0	0.5	N/A						
	JB Indirect	0	2	1	4	1.75	1	0	0	0	0.25							
	No-jump	0	2	0	0	0.5	0	1	1	2	1	5	5	1	1	0	0	2
Total		50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50

Note: FP and HP refer to the final price and hypothetical price, respectively. Similarly, OB, UB, and JB refer to overbidding, underbidding, and jump bidding, respectively. The same abbreviations apply to all other figures and tables hereafter.

By carefully observing Figures 1 and 2, all three types of overbidding and underbidding specified above are observed. First, there are a few instances in which over/underbidding has been *directly* driven by jump bidding; i.e., the initial price is the same as the final price, which is strictly positive (e.g., Round 9 in Pair 1 of Matching Group 3, *Baseline* for overbidding/Round 1 in Pair 2 of Matching Group 3, *Baseline* for underbidding). In these observations, one of the bidders decides to quit immediately after observing the jump bid made by his/her opponent. Second, over/underbidding is sometimes *indirectly* driven by jump bidding; i.e., the initial price is positive, but the final price is strictly higher than the initial price (e.g., Round 3 in Pair 3 of Matching Group 2, *Baseline* for overbidding/Round 3 in Pair 2 of Matching Group 2, *Baseline* for underbidding). Third, over/underbidding occurs even when the initial price is 0 (e.g., Round 1 in Pair 2 of Matching Group 1, *Binary*

[26]We define overbidding (underbidding) as the case in which the final price is strictly higher (lower) than the hypothetical price $+1$ (-1). This definition allows a small degree of mistakes in bidding behavior. All qualitative findings in this paper will be preserved even when we allow no degree or a larger degree (± 2 and ± 3) of mistakes.

for overbidding/Round 5 in Pair 2 of Matching Group 1, *Binary* for underbidding). Table 2 reports the frequencies of different types of overbidding and underbidding for each treatment. In both the *Baseline* and *Binary* treatments, overbidding was more frequently observed than underbidding.

The bidding behaviors in the *Baseline* and *Binary* treatments are qualitatively the same. The Mann-Whitney test (MW test, hereafter) reveals that we cannot reject the null hypothesis that the frequencies of overbidding not driven by jump bidding in the two treatments are not significantly different (two-sided, p-value $= .2454$).[27] The same tests for all other pairwise comparisons (for the frequencies of overbidding directly driven by jump bidding, those indirectly driven by jump bidding, the frequencies of underbidding directly driven by jump bidding, those indirectly driven by jump bidding, and those not driven by jump bidding) generate qualitatively the same result with p-values ranging between .1635 and 1.[28]

In the *Baseline* treatment, the frequencies of overbidding directly induced by jump bidding are not significantly different from those of underbidding directly induced by jump bidding (paired Wilcoxon test, Wilcoxon test hereafter, p-values $> .3458$). However, in the *Binary* treatment, the frequencies of overbidding induced both directly and indirectly by jump bidding are marginally higher than those of underbidding induced by jump bidding (Wilcoxon test, p-values $= .0975$).

Result 1. *In the Baseline and Binary treatments, jump bidding induces not only overbidding but also underbidding. In the Binary treatment, the frequency of overbidding induced both directly and indirectly by jump bidding is higher than that of underbidding. The difference is marginally significant.*

We now present data from the *No-jump* treatment in Figure 3. The figure contains six panels, with each panel referencing a different matching group. There is no dark bar for the initial price because a positive initial bid is not allowed in the treatment. One of the main features emerging from the data is that a substantial degree of overbidding is observed (111 out of 300 observations), while some but very little underbidding is also

[27]This is due to the high variance in both frequencies, although the average frequency of 8.25 from the *Binary* treatment seems to be much larger than the 3.25 from the *Baseline* treatment.

[28]All nonparametric test results reported in the paper are from two-sided tests unless stated otherwise.

observed (12 out of 300).[29] Table 2 also provides a summary of the bidding behavior in the *No-jump* treatment.

It is evident that overbidding is more prevalent in the *No-jump* treatment than in the other two treatments.[30] The MW test reveals that the frequency of overbidding in the *No-jump* treatment is significantly higher than the frequency of overbidding not driven by jump bidding from the *Baseline* treatment (p-value = .0095) and from the *Binary* treatment (p-value = .0543). However, there is no significant difference ($p > 0.3027$; for all MW tests for pairwise comparisons) in the three treatments in terms of the frequency of underbidding not driven by jump bidding.

Result 2. *In all three treatments, a substantial degree of overbidding not driven by jump bidding was observed. The observed frequency of overbidding in the No-jump treatment was significantly higher than those in the other two treatments.*

4.2 Revenue Analysis

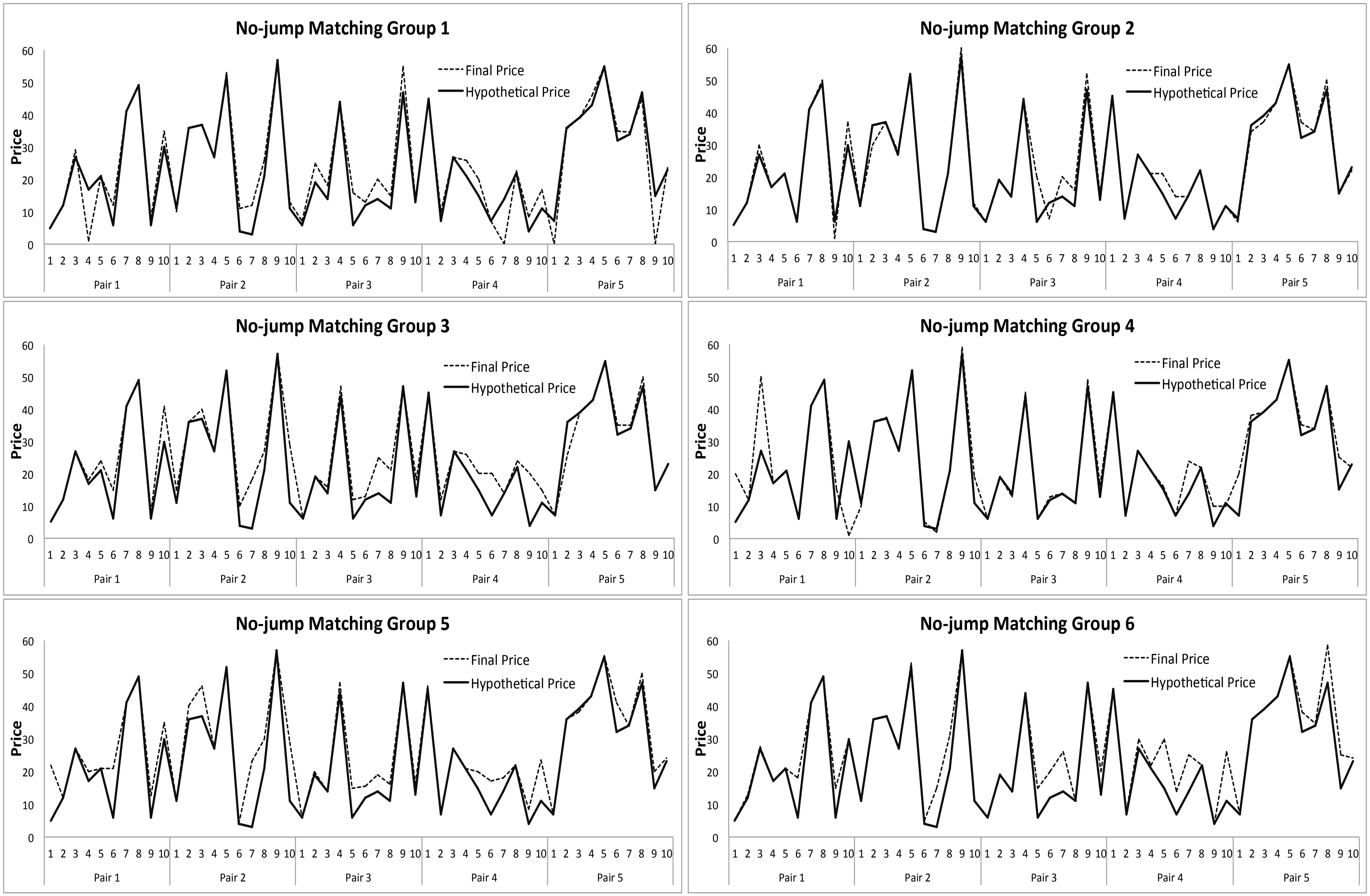
Table 3 reports the result from a decomposition of the difference between the hypothetical and the actual revenues. Specifically, we decompose the contribution of the difference into six parts: overbidding directly/indirectly/not driven by jump bidding and underbidding directly/indirectly/not driven by jump bidding. The *hypothetical* revenue is simply the sum of the hypothetical prices from all pairs and all rounds.

A few findings emerge from the analysis of the *Baseline* and *Binary* treatments. First, the actual revenues are higher than the hypothetical revenues, although the difference is

[29] The overbidding observed in the first two sessions (four matching groups) of the *No-jump* treatment was an unexpected surprise to us, and we held an additional session (two matching groups) for a robustness check.

[30] It is possible that the higher frequency of overbidding observed in the *No-jump* treatment than in the other two treatments may be a consequence of the simplicity of the game the subjects played in the *No-jump* treatment. Without the initial-bidding stage, there is no concern about information transmission, and the remaining game has a truthfully revealing weakly dominant strategy equilibrium. The equilibrium is quite intuitive and easy to understand but does not allow room for strategic interaction. As a result, subjects may have easily become bored, and/or those who have a lower value (and realize that there is nothing they can do to win) may feel a sense of injustice, which may provoke nonequilibrium behavior affected by other-regarding preferences. Note, however, that it is not our primary concern to understand why we have asymmetric overbidding outcomes. In the next section, we will instead propose a new experimental design to control for overbidding.

Figure 3: Bidding Behaviors – *No-jump* Treatment



not statistically significant (Wilcoxon tests, p-values $= .125$ for both cases). Second, the significant proportion of the revenue increase originates from the positive contributions from overbidding directly and indirectly induced by jump bidding. Furthermore, there is a nonnegligible amount of revenue increases from overbidding not driven by jump bidding. Third, underbidding has a negative but minor (relative to overbidding) effect on revenue. The actual revenues from the two treatments are statistically the same (MW test, p-value $= .4857$).

The bottom panel of Table 3 shows that the effect of overbidding on revenue in the *No-jump* treatment is positive and substantial. More important, the increased revenue as a result of overbidding in the *No-jump* treatment is significantly larger than the increased revenue as a result of overbidding not driven by jump bidding in the other two treatments. Consequently, the MW test indicates that we cannot reject the null hypothesis that the actual revenue from the *No-jump* treatment is the same as that from the *Baseline* treatment (p-value $= .4542$) and that from the *Binary* treatment (p-value $= 1$).

Result 3. *The actual revenue from the No-jump treatment is not different from the actual revenues from the Baseline and Binary treatments.*

On the one hand, the result from the revenue comparison already suggests that existing theory cannot explain our data because all existing papers predict that jump bidding reduces the seller's revenue.[31] On the other hand, there is a possibility that the revenue ranking may be misleading due to overbidding behavior, which may create heterogeneous effects on revenues across different treatments. Thus, it is necessary to more carefully study overbidding behavior and determine whether we can isolate the effect of jump bidding on sellers' revenue from that of overbidding not driven by jump bidding.

[31] An alternative hypothesis would be that jump bidding is driven by bidders' impatience (Isaac, Salmon and Zillante [23]). However, our experimental result presented in the next section that there is substantial amount of revenue increases from jump bidding, which has some signaling value, rejects this alternative hypothesis.

Table 3: Total Revenue Decomposition

Baseline	*Hypothetical*	Overbidding JB Direct	Overbidding JB Indirect	Overbidding No-jump	Underbidding JB Direct	Underbidding JB Indirect	Underbidding No-jump	*Actual*
Matching Group 1	1,163	+21	+11	+1	-3	-5	-3	1,185
Matching Group 2	1,163	+36	+26	+27	-1	-44	-3	1,204
Matching Group 3	1,163	+3	+236	+72	-4	-4	-1	1,465
Matching Group 4	1,163	+11	+4	+87	0	-25	-7	1,233
Mean	1,163	+17.75	+69.25	+46.75	-2	-19.5	-3.5	1,271.75

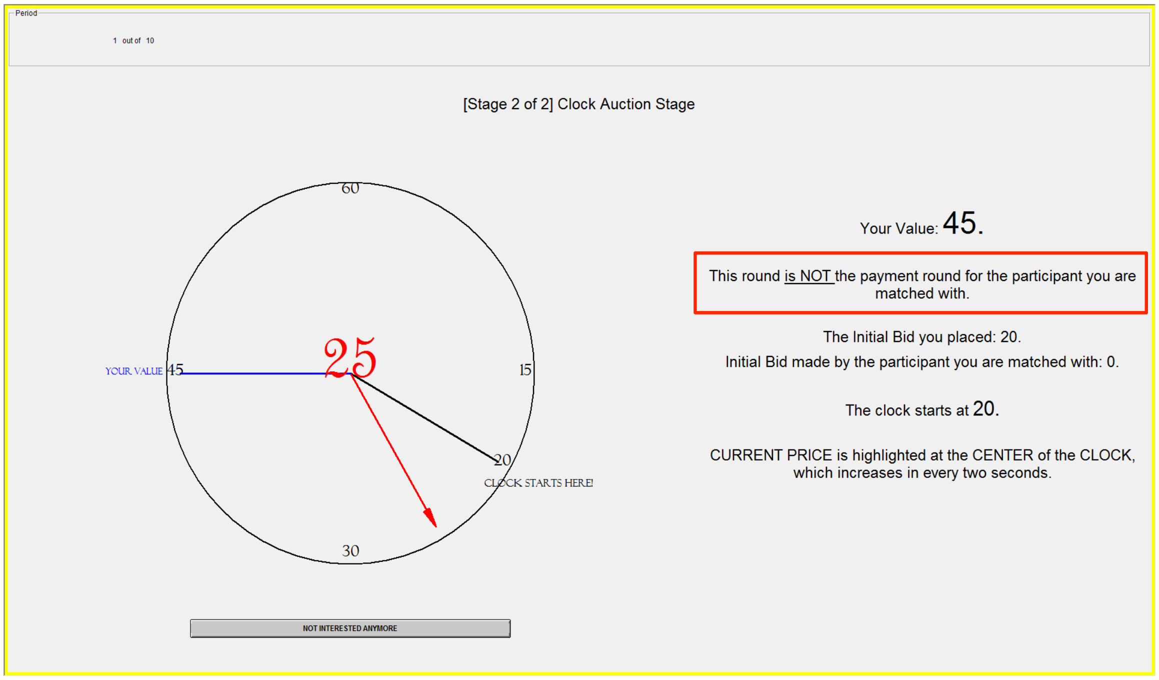
Binary	*Hypothetical*	Overbidding JB Direct	Overbidding JB Indirect	Overbidding No-jump	Underbidding JB Direct	Underbidding JB Indirect	Underbidding No-jump	*Actual*
Matching Group 1	1,163	+34	+65	+55	-32	-6	0	1,279
Matching Group 2	1,163	+51	+38	+95	-1	0	-7	1,339
Matching Group 3	1,163	+22	+33	+21	0	0	-22	1,217
Matching Group 4	1,163	+8	+68	+93	0	-1	-11	1,320
Mean	1,163	+28.75	+51	+66	-8.25	-1.75	-10	1,288.75

No-jump	*Hypothetical*	Overbidding	Underbidding	*Actual*
Matching Group 1	1,163	+109	-55	1,217
Matching Group 2	1,163	+69	-23	1,209
Matching Group 3	1,163	+173	-10	1,326
Matching Group 4	1,163	+110	-34	1,239
Matching Group 5	1,163	+188	0	1,351
Matching Group 6	1,163	+163	0	1,326
Mean	1,163	+135.34	-20.34	1,278

5 A New Experimental Design and Its Results

Our exit survey result indicates that overbidding not driven by jump bidding in our treatments is mainly induced by spiteful motives.[32] We thus design two new treatments, *Binary-II* and *No-jump-II*, parallel to the *Binary* and *No-jump* treatments, with an additional experimental control. Two sessions for each treatment with 20 subjects for each session were conducted. Recall that in our treatments, we randomly selected one round (out of 10) to calculate the final payment for each subject. In the new design, which we name the *Amended Random Payment* (*ARP*) design, we informed each individual whether the current round is the payment round for his/her opponent (see Figure 4).[33] Throughout the auction, however, each individual never knew whether the current round was his/her payment round. The purpose of the new design is to fully control for or discard any kind of other-regarding preferences that may affect subjects' bidding behavior.[34] We also carefully revised the experimental instructions, replacing any words that might provoke a joy of winning and/or spitefulness (such as win/lose and opponent) with more

Figure 4: Z-tree Screen Shot – *Binary-II* Treatment



neutral terms.

There are a few other possible approaches described in the literature to control for other-regarding preferences. One can consider a longer time horizon, as in Kagel,

[32]Here are a few selected responses to the question "Suppose that your value is 10. Briefly describe your behavior in the auction" in our exit survey.
"Wait until the price go[es] higher, maybe this is to prevent others [from] earn[ing] more. "
"Since my value is quite small, I would want to have some risk. I should stay in the auction even if the current value [price] exceed[s] my value. I would quit the auction when the current value [price] reaches 15. As 15 is still a small number, I would assume that the opponent has a value bigger than 15. But as I quit after the current value [price] exceed[s] my value, the opponent is going to earn less. "
"10 is small number, which has almost no chance of winning the auction, so what I am going to do is to minimize my opponent's gain. Usually, I will wait until the clock reaches about 18-20 and opt out of the auction because in this case there is an over 80% chance that my opponent will have a larger value, so I will bet he/she has a value not smaller than 25."

[33]The current design of the *ARP* scheme applies only to the two-bidder setting. However, as suggested by an anonymous referee, the *ARP* design can be easily extended to multibidder settings. For instance, with more than two bidders, we can inform every bidder whether the current round is his/her most competitive opponent's payment round, as such an opponent is most likely to win the auction.

[34]Specifically, suppose that an individual's utility consists of two terms as follows:

$$U_i = \alpha_i \cdot u_i + \beta_i \cdot v_i \tag{2}$$

where u_i denotes the utility that comes from the material payoff and $v_i = f(u_{-i})$ denotes the utility coming from other-regarding preferences. $\alpha_i > 0$ and $\beta_i > 0$ measure the respective importance of the two terms. Under the *ARP* design, if an individual i is informed that the current round is not the payment round of his/her opponent, then $\beta_i = 0$ such that $U_i = u_i$.

Harstad, and Levin [26], or consider inviting more experienced subjects, as in Garratt, Walker and Wooders [16]. One can also design a game such that a subject plays against a fictitious player such as a robot with a particular strategy or against prior human players (e.g., Cason and Sharma [7], Johnson, Camerer, Sen, and Rymon [24]). We believe that our *ARP* method may have some advantages. First, it provides a way to eliminate other-regarding preferences of human subjects without relying on a fictitious player. Second, it allows us to make a direct comparison between data with and without the concerns of other-regarding preferences for the same set of human subjects. Third, our method is very simple to implement and can be applied to a broad range of games. Fourth, our method is less restrictive with respect to practical concerns such as inviting human subjects who satisfy certain conditions (e.g., experience) and keeping the length of a session reasonably short.

5.1 Bidding Behavior

Figures 5 and 6 present the experimental data from the two treatments. Tables 4, 5 and 6 report summaries of bidding frequencies, data from the opponent's payment rounds, and data from the opponent's nonpayment rounds, respectively. One noticeable difference from the treatments in the previous sections is that overbidding is observed significantly less frequently in the *No-jump-II* treatment than in the *No-jump* treatment (8.25 vs. 18.5, MW test, p-value $= .0139$).

To understand the relationship between overbidding and the spite motive, it is useful to focus on the data from the opponent's payment rounds presented in Table 5. Two features emerge from the data. First, a significant degree of overbidding was observed in both treatments. For each matching group, there were 8 rounds in which a spite opportunity existed (i.e., when a subject with a lower value was informed that the current round is the payment round of the opponent), and subjects indeed overbid frequently (on average, 3.5 times in both the *Binary-II* and *No-jump-II* treatments). The same information can be found in Figures 5 and 6, where the solid circles highlight the instances in which the final price is strictly higher than the hypothetical price when the current round is the

Table 4: Bidding Frequencies – All Data

		Binary-II					*No-jump-II*				
Matching Group		1	2	3	4	Mean	1	2	3	4	Mean
FP = HP (±1)		27	36	26	26	28.75 (33)	31	42	41	31	36.25 (29.5)
OB	JB Direct	6	2	6	5	4.75 (3.75)	N/A				
	JB Indirect	7	4	3	6	5 (3.25)					
	No-Jump	3	5	5	7	5 (8.25)	8	7	8	10	8.25 (18.5)
UB	JB Direct	0	2	2	1	1.25 (0.5)	N/A				
	JB Indirect	4	1	2	2	2.25 (0.25)					
	No-Jump	3	0	6	3	3 (1)	11	1	1	9	5.5 (2)
Total		50	50	50	50	50	50	50	50	50	50

Note: Numbers inside brackets are corresponding values from the *Binary* and *No-jump* treatments.

Table 5: Bidding Frequencies – Payment Rounds

		Binary-II					*No-jump-II*				
Matching Group		1	2	3	4	Mean	1	2	3	4	Mean
FP = HP (±1)		4	6	4	4	4.5	4	8	5	3	5
OB	JB Direct	1	0	2	0	0.75	N/A				
	Else	4	4	2	4	3.5	3	2	4	5	3.5
UB	JB Direct	0	0	0	0	0	N/A				
	Else	1	0	2	2	1.25	3	0	1	2	1.5
Total		10	10	10	10	10	10	10	10	10	10

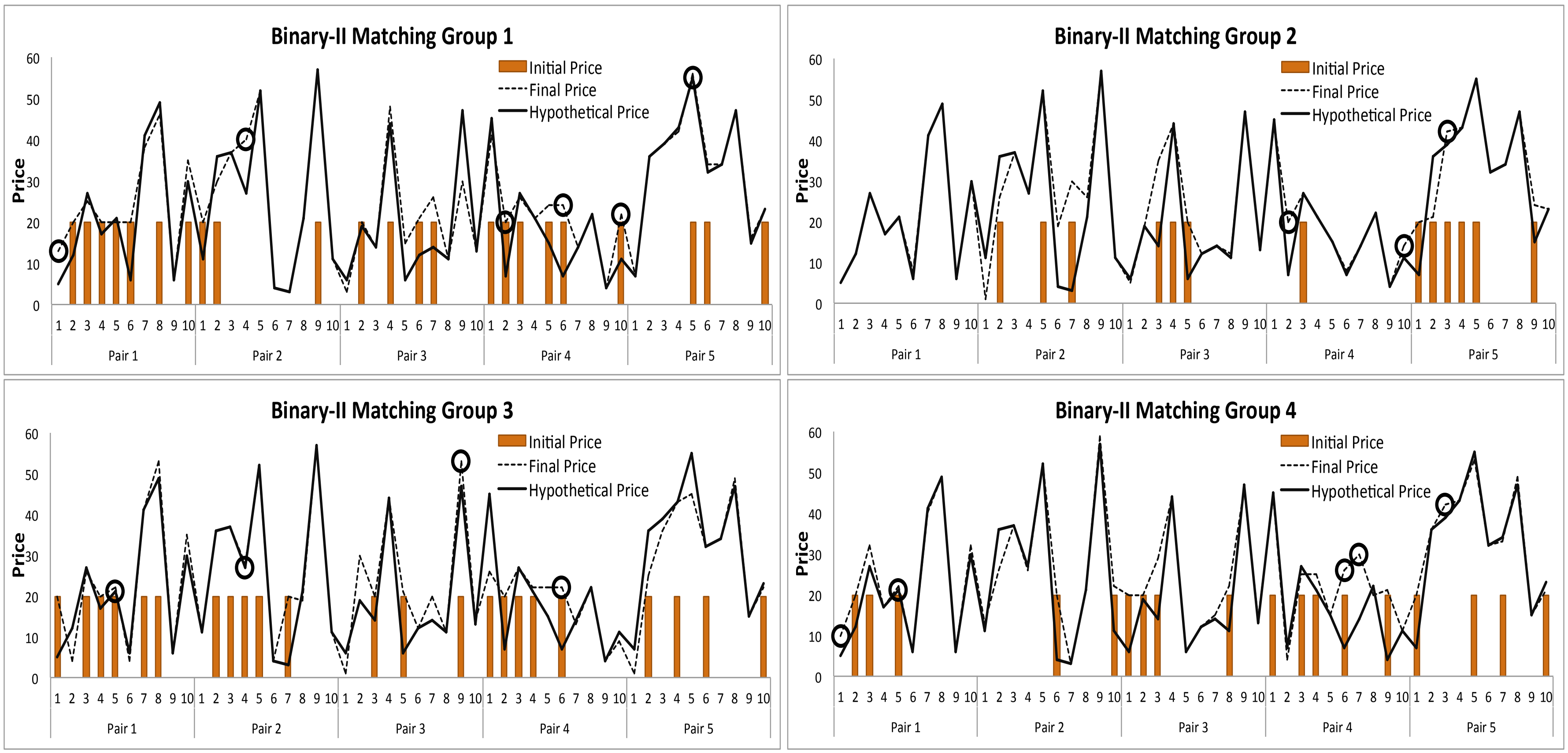
Table 6: Bidding Frequencies – Non-payment Rounds

		Binary-II					*No-jump-II*				
Matching Group		1	2	3	4	Mean	1	2	3	4	Mean
FP = HP (±1)		23	30	22	22	24.25	27	34	36	28	31.25
OB	JB Direct	5	2	4	5	4	N/A				
	JB Indirect	5	2	1	5	3.25					
	Else	1	3	5	4	3.25	5	5	4	5	4.75
UB	JB Direct	0	2	2	1	1.25	N/A				
	JB Indirect	4	1	2	1	2					
	Else	2	0	4	2	2	8	1	0	7	4
Total		40	40	40	40	40	40	40	40	40	40

payment round of the opponent.[35] Second, there is no significant difference in bidding

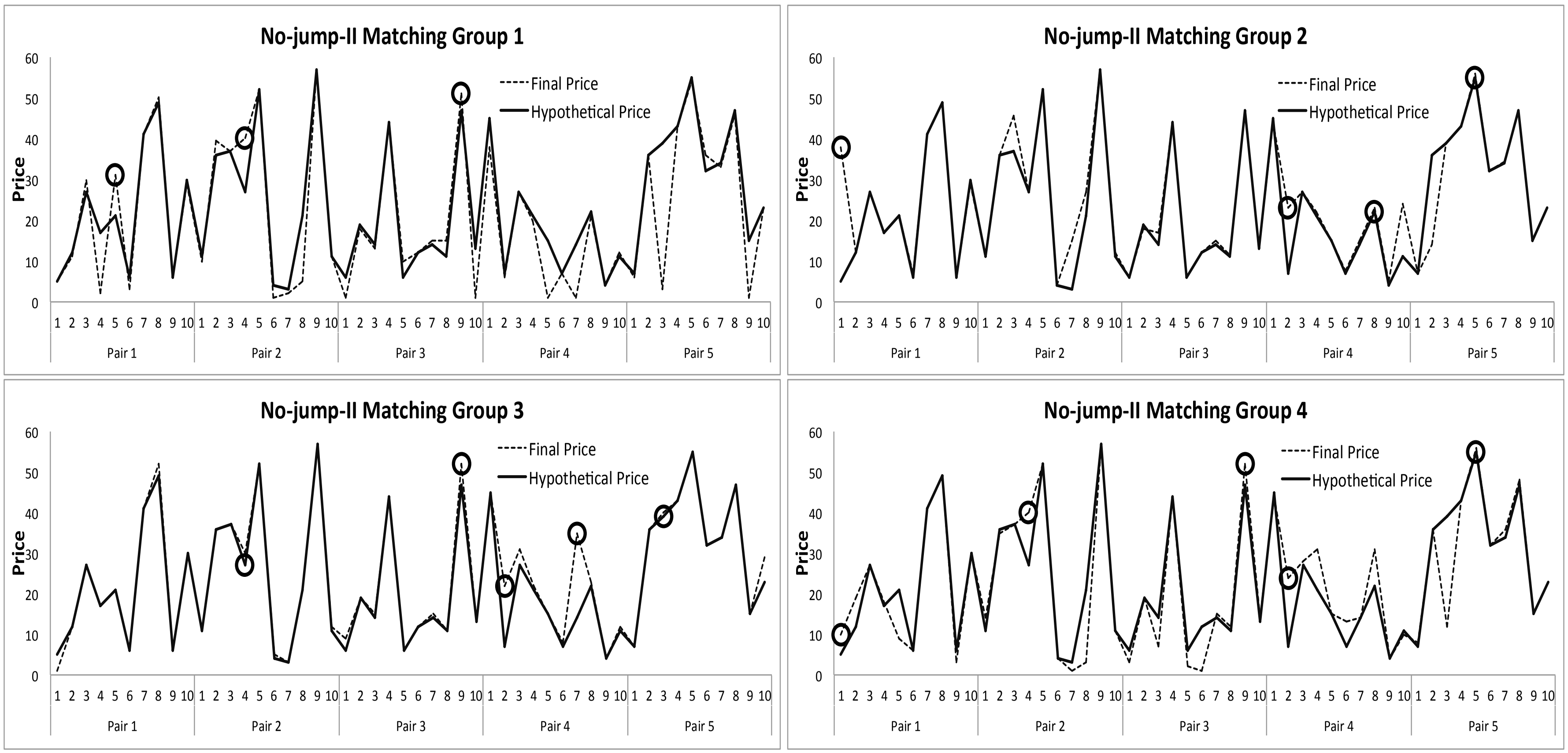
[35] In total, there are 18 and 17 solid circles in Figures 5 and 6, respectively, indicating that the average frequency of overbidding induced by the spite motive is 4.5 and 4.25 in the two treatments. Four such ob-

Figure 5: Bidding Behaviors – *Binary-II* Treatment



Note: The solid circles indicate that overbidding is observed when the current round is the opponent's payment round.

Figure 6: Bidding Behaviors – *No-jump-II* Treatment



Note: The solid circles indicate that overbidding is observed when the current round is the opponent's payment round.

behavior conditional on the current round being the payment round of the opponent in the two treatments. An MW test reveals that the frequency of overbidding not driven by jump bidding in *Binary-II* is not significantly different from the frequency of overbidding in the *No-jump-II* treatment (p-value = 1). Similarly, the frequency of underbidding not driven by jump bidding in the *Binary-II* treatment is not significantly different from the frequency of underbidding in the *No-jump-II* treatment (p-value = .8809). These observations suggest that our *ARP* design allows us to identify spite-driven overbidding and to successfully separate it from jump-driven overbidding.

We now focus on the data from the opponent's nonpayment rounds presented in Table 6. Admittedly, overbidding and underbidding led by mistakes and misunderstandings are still unavoidable.[36] However, such over/underbidding behavior seems to be well controlled, as no significant difference in the frequency of over/underbidding exists (MW test, p-value = .1354 for overbidding and = .7702 for underbidding) between the two treatments. Hence, the only qualitative difference between the two treatments in terms of the bidding behavior conditional on the current round being one of the nonpayment rounds of the opponent is the presence/absence of the over/underbidding directly and indirectly induced by jump bidding.[37] The frequency of overbidding induced by jump bidding is significantly higher than that of underbidding in the *Binary-II* treatment (MW test, p-value = .0530).

Result 4. *1) In the Binary-II treatment, jump bidding induces not only overbidding but also underbidding. Conditional on the current round being one of the non-payment rounds of the opponent, 2) the frequency of overbidding induced both directly and indirectly by jump bidding is higher than that of underbidding in the Binary-II treatment, and 3) a non-negligible degree of overbidding not driven by jump bidding is observed in both the Binary-II and No-jump-II treatments,*

servations in the *Binary-II* treatment and 3 in the *No-jump-II* treatment have a final price just one unit higher than the hypothetical price and thus are not included in the overbidding category so that the reported average frequencies in Table 5 turn out to be 3.5 for both treatments.

[36] Our exit survey result reveals that overbidding was committed by mistake or by misunderstanding by subjects who believed that they could still spite their opponent even in the nonpayment round. Regarding underbidding, there were three individuals who reported that they gave up (and bid 0) whenever their value was smaller than a certain level.

[37] The jump-bidding equilibrium (σ_1^*, σ_2^*) with the following slight modification predicts the observed overbidding indirectly induced by jump bidding well: First, high-value bidders ($v_i \geqslant v^*$) jump to $k(v^*) - \delta$ with $\delta > 0$; low-value bidders ($v_i < v^*$) do not jump. Second, all bidders remain after observing the jump $k(v^*) - \delta$. Third, low-value bidders quit at price $k(v^*)$; high-value bidders remain after price $k(v^*)$. Thus, when $v_i \in [0, k(v^*))$, the overbidding indirectly induced by jump bidding is expected.

with no significant difference across the treatments.

5.2 Revenue Analysis

Table 7: Total Revenue Decomposition – All Data

		Overbidding			Underbidding			
Binary-II	*Hypothetical*	JB Direct	JB Indirect	No-jump	JB Direct	JB Indirect	No-jump	*Actual*
Matching Group 1	1,163	+48	+70	+31	-1	-16	-24	1,271
Matching Group 2	1,163	+27	+60	+40	0	-25	-12	1,253
Matching Group 3	1,163	+54	+43	+32	0	-33	-39	1,220
Matching Group 4	1,163	+52	+83	+34	0	-25	-12	1,306
Mean	1,163	+45.25	+64	+34.25	-0.25	-24.75	-21.75	1,262.5
No-jump-II	*Hypothetical*		Overbidding			Underbidding		*Actual*
Matching Group 1	1,163		+48			-151		1,060
Matching Group 2	1,163		+99			-25		1,237
Matching Group 3	1,163		+69			-4		1,228
Matching Group 4	1,163		+85			-90		1,158
Mean	1,163		+75.25			-67.5		1,170.75

Tables 7 and 8 report the total revenue decomposition result based on all data and based on the data from the opponent's nonpayment rounds, respectively. Table 7 shows that jump bidding contributes significantly and positively to revenue. Consequently, the actual revenue from the *Binary-II* treatment is larger than that from the *No-jump-II* treatment, although the difference is not significant (MW test, p-value $=$ .1142). The magnitude of the revenue increase induced by overbidding is larger in the *No-jump-II* treatment than in the *Binary-II* treatment (MW test, p-value $=$ .0142). Moreover, the magnitude of the revenue decrease induced by underbidding in the *No-jump-II* treatment is not significantly different from that in the *Binary-II* treatment (MW test, p-value $=$ .5577).

The result from the nonpayment round data presented in Table 8 confirms and reinforces the main finding from all the data. First, the Wilcoxon test reveals that the actual revenue from the *Binary-II* treatment is higher than the hypothetical revenue, albeit only insignificantly (p-value $=$ 0.125). Second, the MW test shows that the actual revenue

Table 8: Total Revenue Decomposition – Non-payment Rounds

		Overbidding			Underbidding			
Binary-II	*Hypothetical*	JB Direct	JB Indirect	No-jump	JB Direct	JB Indirect	No-jump	*Actual*
Matching Group 1	**929**	+35	+41	+9	0	-16	-7	991
Matching Group 2	**929**	+27	+48	+23	0	-25	-12	990
Matching Group 3	926	+26	+20	+32	0	-33	-25	946
Matching Group 4	926	+52	+63	+10	0	-6	-13	1,032
Mean	927.5	+35	+43	+18.5	0	-20	-14.25	989.75

No-jump-II	*Hypothetical*	Overbidding	Underbidding	*Actual*
Matching Group 1	926	+21	-86	861
Matching Group 2	926	+48	-25	949
Matching Group 3	926	+23	0	949
Matching Group 4	926	+38	-51	913
Mean	926	+32.5	-40.5	918

Note: Because of a programming mistake in the first session of *Binary-II* treatment, we had the payment round of one subject in each matching group different from that of other sessions. As a result, the hypothetical revenue in matching groups 1 and 2 of the *Binary-II* treatment is 929, slightly higher than the 926 in the other sessions.

from the *Binary-II* treatment is higher than the actual revenue from the *No-jump-II* treatment (p-value $= .110$), again with a marginal significance. However, we cannot reject the null hypothesis that the actual revenue from the *No-jump-II* treatment is not significantly different from the hypothetical revenue (Wilcoxon test, p-value $= 1.000$).

Result 5. *In our ARP design, 1) the actual revenue from the No-jump-II treatment is not different from the hypothetical revenue, and 2) the actual revenue from the Binary-II treatment is (only insignificantly) higher than the hypothetical revenue and the actual revenue from the No-jump-II treatment.*

The statistical insignificance we have in the above result may be due to the small sample size (4 matching groups). We thus conduct an additional regression analysis using the 1st round individual-level observations to understand the revenue effect of jump bidding.[38] We regress the final price paid by the winner on the following variables: v_i, a variable for bidder i's value; $\mathbb{I}\{Treatment\}$, an indicator variable that takes value 1 in the

[38] We thank the anonymous referee for making this useful suggestion.

Jump-II treatment; $\mathbb{I}\{OpponenPaymentRound\}$, an indicator variable that takes value 1 if it is the opponent's payment round; $\mathbb{I}\{InitialBid\}$, an indicator variable that takes value 1if the jump bid of 20 is chosen as an initial bid; $\mathbb{I}\{OpponentInitialBid\}$, an indicator variable that takes value 1 if the jump bid of 20 is chosen by the opponent. For every indicator variable, we slightly abuse notation and use the name inside the indicator function to denote the variable itself.

Table 9: Linear Regression Model with 1st Round Data

	(1)	(2)
Constant	.2.9893	3.7982
	(2.8848)	(2.7391)
Treatment	.3500	−4.6307
	(2.6841)	(2.9128)
v_i (Value)	.5130***	.4795***
	(.0821)	(.0795)
Opponent's Payment Round	7.0208	6.9027
	(4.7911)	(4.4797)
Initial Bid	–	.4584
	–	(.2386)
Opponent Initial Bid	–	.7868***
	–	(.2292)
No. of Observations	80	80

Note: Standard errors are in parentheses. *** indicates significance at the 0.1% level, ** significance at the 1% level, and * significance at the 5% level.

Table 9 reports the regression result. The column (1) shows that the final price (equivalently, the revenue) is only insignificantly higher in the *Jump-II* treatment than in the *No-Jump-II* treatment when controlling for the value and the opponent's payment round. With two more regressors, the column (2) further indicates that the observed revenue increase originates from the jump-bid made by an individual and that made by the opponent while only the latter is significant. This result is straightforward to interpret. On one hand, one's own jump-biding crucially depends on the value the individual has. As a result, when controlling for the value, the effect of individual's own jump-bidding on the revenue is insignificant. On the other hand, the significantly positive effect of the opponent's jump-bidding implies that one needs to pay a higher final price when the opponent jump-bids comparing to the case in which the opponent does not while fixing the value and his own jump-bidding constant. This result essentially answers the question we raised in the introduction. It provides a rationale for why jump bidding is not prohibited in many real auctions.

6 Determinants of Jump bidding

In this section, we investigate major determinants of jump bidding. Our theory suggests that jump bidding is a device for a bidder to signal his/her high value. Moreover, Proposition 2 presents a few *sufficient* conditions for the jump-bidding equilibrium regarding players' risk attitudes. We thus regress $\mathbb{I}\{J_i = 1\}$, an indicator variable that takes value 1 if bidder i jumps and zero otherwise, on the following variables: v_i, a variable for bidder i's value; $\mathbb{I}\{Baseline\}$, an indicator variable that takes value 1 in the *Baseline* treatment; $\mathbb{I}\{ARP\}$, an indicator variable that takes value 1 in the *Binary-II* treatment; $\mathbb{I}\{Aversion\}$, an indicator variable that takes value 1if an individual's identified risk attitude is aversion; $\mathbb{I}\{Neutral\}$, an indicator variable that takes value 1 if an individual's identified risk attitude is neutral; $\mathbb{I}\{Loving\}$, an indicator variable that takes value 1 if an individual's identified risk attitude is loving; $\mathbb{I}\{IARA\}$, an indicator variable that takes value 1 if an individual's identified absolute-risk attitude is IARA; $\mathbb{I}\{CARA\}$, an indicator variable that takes value 1 if an individual's identified absolute-risk attitude is CARA; $\mathbb{I}\{DARA\}$, an indicator variable that takes value 1 if an individual's identified absolute-risk attitude is DARA; $\mathbb{I}\{Female\}$, an indicator variable that takes value 1 if an individual is a female; and t, a variable for the period. For every indicator variable, we slightly abuse notation and use the name inside the indicator function to denote the variable itself.

Table 10 presents the results from the regression analysis using the data from our three treatments with jump bidding (*Baseline*, *Binary*, and *Binary-II*). The first column reports the results from the probit regression. The second and third columns report the results from the logit and linear regression models, respectively. Regardless of the regression specification considered, we have consistent results as follows. First, the coefficient for the bidder's value v_i is positive and significant, which confirms that jump bidding is a credible signaling device. Second, the coefficient for the *Baseline* treatment is negative and significant. This indicates that jump bidding is more prevalent in the *Binary*

Table 10: Probit, Logit, and Linear Probability Models

	(1)	(2)	(3)
Constant	.7238***	1.2047***	.7296***
	(.013)	(.3231)	(.0506)
v_i (*Value*)	.0140***	.0241***	0.0036***
	(.0026)	(.0045)	(.0007)
Baseline	−1.6942***	−2.8544***	−0.5396***
	(.1094)	(0.1964)	(.0280)
ARP	.2068*	.3399*	.0463*
	(.1105)	(.1990)	(.0281)
Aversion	−.2479	−.4022	−.0699
	(.1974)	(.3312)	(.0528)
Neutral	.3602	.6931	.0969
	(.2606)	(.4420)	(.0700)
Loving	−.3618	−.5889	−.0883
	(.2493)	(.4212)	(.0654)
IARA	.0703	.0588	.0187
	(.2419)	(.4097)	(.0650)
CARA	−.1618	−.3383	−.0400
	(.2445)	(.4148)	(.0653)
DARA	.3344	.5495	.0911
	(.2186)	(.3675)	(.0584)
t (*Period*)	−.1025***	−.1756***	−.0264***
	(.0152)	(.0270)	(.0040)
Female	.1464	.2809*	.0436*
	(.0912)	(.1594)	(.0243)
No. of Observations	1,200	1,200	1,200

Note: Standard errors are in parentheses. *** indicates significance at the 0.1% level, ** significance at the 1% level, and * significance at the 5% level.

treatment than in the *Baseline* treatment. This result reflects the fact that there is a unique jump-bidding equilibrium in the *Binary* treatment. Third, the coefficient for *ARP* is positive and marginally significant. This indicates that our ARP design successfully isolates other confounding factors, including other-regarding preferences. Fourth, the coefficient for *period* is negative and significant. Subjects seem to learn not to jump when gaining more experience. Fifth, female students are more likely to jump. Finally, the elicited risk attitudes are all insignificant. This indicates that the conditions regarding risk attitudes provided by Proposition 2 are sufficient but not necessary conditions for the existence of a jump-bidding equilibrium. Moreover, it also reveals that measurement error may be serious in the elicitation of risk attitudes.

7 Conclusion

Jump bidding is frequently observed in real-life auctions. Although several papers (e.g., Avery [2] and Daniel and Hirshleifer [13]) have provided convincing analyses on this phenomenon, previous works suggest that sellers' revenue decreases when jump bidding occurs, a finding in sharp contrast to the fact that jump bidding is allowed in real-life auctions (e.g., Sotheby's auctions and FCC spectrum auctions). In this paper, our auction experiments demonstrate that sellers' revenue increases when jump bidding occurs. The result is qualitatively consistent with the theory of jump bidding we proposed.

Finally, because we consider different settings than other researchers, we wish to emphasize that our results do *not* imply that the previous papers are incorrect.[39] Rather, a complete picture of jump bidding has yet to be seen. Thus, this paper complements previous studies by using experiments and a particular theory to identify the positive revenue effect of jump bidding.

[39] For example, we adopt the IPV setup, whereas Avery [2] adopts the common-value setup; we consider risk-averse bidders, whereas Daniel and Hirshleifer [13] and Avery [2] consider risk-neutral bidders.

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Appendix

A Figures and Tables

Table 11: Values Used in the Experiment

Subject	Round	1	2	3	4	5	6	7	8	9	10
1	Value	5	24	37	44	55	18	44	22	4	11
	Pair	1	4	2	3	5	1	5	3	4	2
2	Value	47	44	39	43	52	12	41	45	6	24
	Pair	4	1	5	5	2	3	4	4	1	4
3	Value	25	36	27	29	6	32	60	58	59	16
	Pair	1	2	1	1	3	5	1	5	2	3
4	Value	7	36	58	17	51	20	14	27	15	13
	Pair	5	5	4	1	3	2	4	2	5	3
5	Value	34	36	46	42	21	40	47	11	57	41
	Pair	2	3	2	2	1	3	3	3	2	1
6	Value	49	53	43	48	23	6	3	21	29	11
	Pair	5	2	5	3	4	1	2	2	5	4
7	Value	6	12	27	27	31	41	41	49	47	29
	Pair	3	1	4	2	1	5	1	1	3	2
8	Value	9	48	39	60	53	22	44	58	59	23
	Pair	3	5	3	5	2	4	2	1	3	5
9	Value	11	7	14	21	55	4	34	22	19	29
	Pair	2	4	3	4	5	2	5	4	1	5
10	Value	45	19	50	60	15	7	14	47	43	30
	Pair	4	3	1	4	4	4	3	5	4	1

Table 12: Eliciting Risk Attitudes

Row No.	Option A		Option B
	Outcome $A1 = \$Y + h$	Outcome $A2 = \$Y - h$	
1	Prob 35/100	Prob 65/100	Certain Outcome $\$Y$
2	Prob 40/100	Prob 60/100	
3	Prob 45/100	Prob 55/100	
4	Prob 50/100	Prob 50/100	
5	Prob 55/100	Prob 45/100	
6	Prob 60/100	Prob 40/100	
7	Prob 65/100	Prob 35/100	
8	Prob 70/100	Prob 30/100	
9	Prob 75/100	Prob 25/100	
10	Prob 80/100	Prob 20/100	
11	Prob 85/100	Prob 15/100	
12	Prob 90/100	Prob 10/100	

Figure 7: Average Frequency of Safe Choices (Option B)

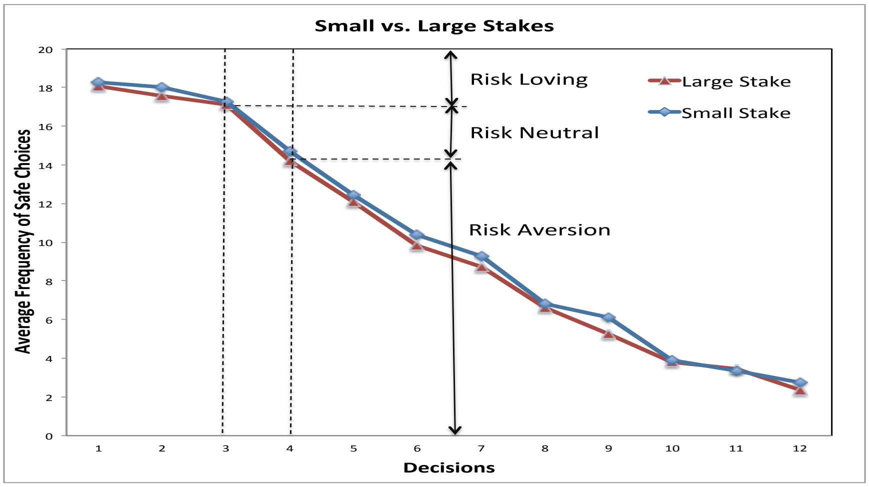


Table 13: Elicited Risk Attitudes - Individual Level

		Risk Attitude				Absolute Risk Aversion			
		Aversion	Neutral	Loving	Unidentified	Increasing	Constant	Decreasing	Unidentified
Baseline	Session 1	10	1	3	6	4	6	7	3
	Session 2	16	1	1	2	4	11	3	1
Binary	Session 1	12	2	3	3	5	8	6	1
	Session 2	14	2	0	4	3	9	4	4
No-jump	Session 1	14	2	1	3	5	9	3	3
	Session 2	14	2	1	3	2	12	4	2
	Session 3	11	3	3	3	4	6	7	3
Binary-II	Session 1	16	2	0	2	4	10	5	1
	Session 2	13	1	1	5	5	6	5	4
No-jump-II	Session 1	9	2	4	5	2	11	4	3
	Session 2	12	2	1	5	3	8	5	4
Total		141 (64.1%)	20 (9.1%)	18 (8.2%)	41 (18.6%)	41 (18.6%)	96 (43.6%)	53 (24.1%)	30 (13.6%)

B Proofs

Proof of Proposition 2

By following (σ_1^*, σ_2^*), the high types (i.e., $v_i, v_{-i} \geqslant v^*$) jump to $k(v^*)$ in stage 1, and the low types (i.e., $v_i, v_{-i} < v^*$) do not jump. I.e., bidders use the jump bid $k(v^*)$ to signal their high values. In the case that $\beta_{-i} = k(v^*)$ and $v_i < v^*$, bidder i infers that $v_{-i} \geqslant v^*$ and expects no chance to win, and hence bidder i quits immediately. For all other cases, bidder i follows the weakly dominant strategy $b_i(\beta_i, \beta_{-i}, v_i) = \max\{\beta_i, \beta_{-i}, v_i\}$ in the clock auction in stage 2. Thus, (σ_1^*, σ_2^*) is a PBE if and only if the bidders' signalling is credible, i.e., the high types prefer "jumping to $k(v^*)$" and the low types prefer "no jump."[40] Define

$$N\left(v_i\right) := \int_0^{v_i} u(v_i - v_{-i})dF(v_{-i}); \tag{3}$$

$$J\left(v_i\right) := F(v^*)u(v_i - k(v^*)) + \int_{v^*}^{\max\{v^*,\, v_i\}} u(v_i - v_{-i})dF(v_{-i}); \tag{4}$$

where $N\left(v_i\right)$ and $J\left(v_i\right)$ are the expected utility of type v_i for "no jump" and "jumping to $k(v^*)$," respectively. We thus have the following lemma.

Lemma 1. *(σ_1^*, σ_2^*) is a PBE if and only if*

$$J\left(v_i\right) - N\left(v_i\right) \begin{cases} \leqslant 0 & \text{if } v_i < v^*; \\ \geqslant 0 & \text{if } v_i \geqslant v^*. \end{cases}$$

Proof of Proposition 2. Define

$$g\left(v_i\right) := u(v_i - k(v^*));$$

$$h\left(v_i\right) := E_{v_{-i} \sim [0, v^*] \text{ with cdf } \frac{F(v_{-i})}{F(v^*)}}\left[u(v_i - v_{-i})\right] = \int_0^{v^*} u(v_i - v_{-i})d\frac{F(v_{-i})}{F(v^*)}.$$

By (21) and (22), we have

$$J\left(v_i\right) - N\left(v_i\right) = \begin{cases} F(v^*) \times [g(v_i) - h(v_i)] - \int_{v^*}^{v_i} u(v_i - v_{-i})dF(v_{-i}) & \text{if } v_i < v^*; \\ F(v^*) \times [g(v_i) - h(v_i)] & \text{if } v_i \geqslant v^*, \end{cases}$$

which implies

$$J\left(v_i\right) - N\left(v_i\right) \begin{cases} \leqslant F(v^*) \times [g(v_i) - h(v_i)] & \text{if } v_i < v^*; \\ = F(v^*) \times [g(v_i) - h(v_i)] & \text{if } v_i \geqslant v^*, \end{cases} \tag{5}$$

because $\int_{v^*}^{v_i} u(v_i - v_{-i})dF(v_{-i}) \geqslant 0$ for every $v_i < v^*$.

[40] According to σ_{-i}^*, if bidder i jumps to any off-equilibrium price in stage 1, bidder $-i$ stays in the auction in stage 2 until the price reaches $-i$'s true value. As a result, jump-bidding is useless for bidder i and he prefers no jump to any off-equilibrium jump in stage 1.

The properties of IARA and CARA (see Matthews [1], p. 638) imply

$$IARA: \; g(v_i) = h(v_i) \implies g'(v_i) > h'(v_i); \tag{6}$$

$$CARA: \; g(v_i) = h(v_i) \implies g'(v_i) = h'(v_i). \tag{7}$$

Note that (26) implies $g(\cdot)$ and $h(\cdot)$ cross at most once. In particular, $g(v^*) = h(v^*)$. Hence,

$$\text{given } IARA: g(v_i) - h(v_i) \begin{cases} < 0, & \text{if } v_i < v^*; \\ = 0, & \text{if } v_i = v^*; \\ > 0, & \text{if } v_i > v^*. \end{cases}; \tag{8}$$

$$\text{given } CARA: g(v_i) - h(v_i) = 0, \forall v_i \in [0,1]. \tag{9}$$

(25), (28) and (9) imply

$$\text{given } IARA \text{ or } CARA: J(v_i) - N(v_i) \begin{cases} \leqslant 0 & \text{if } v_i < v^*; \\ \geqslant 0 & \text{if } v_i \geqslant v^*. \end{cases}$$

Therefore, (σ_1^*, σ_2^*) is a PBE by Lemma 2.■

Proof of Theorem 1

There are three possible events: i) $[v_i < v^* \leqslant v_{-i}$ with $i \in \{1,2\}]$, ii) $[\max\{v_1, v_2\} < v^*]$ and iii) $[v^* \leqslant \min\{v_1, v_2\}]$. In event ii) or iii), the seller gets the same revenue in both (σ_1^*, σ_2^*) and $(\widehat{\sigma}_1, \widehat{\sigma}_2)$. Conditional on event i), the expected revenues in (σ_1^*, σ_2^*) and $(\widehat{\sigma}_1, \widehat{\sigma}_2)$ are $k(v^*)$ and $\int_0^{v^*} v_i d\frac{F(v_i)}{F(v^*)}$, respectively. Given risk-averse bidders, $u(\cdot)$ is strictly concave and

$$u\left[v^* - \int_0^{v^*} v_i d\frac{F(v_i)}{F(v^*)}\right] = u\left[\int_0^{v^*} (v^* - v_i)\, d\frac{F(v_i)}{F(v^*)}\right] > \left[\int_0^{v^*} u(v^* - v_i)\, d\frac{F(v_i)}{F(v^*)}\right] = u(v^* - k(v^*)), \tag{10}$$

where the first equality follows from $\int_0^{v^*} d\frac{F(v_i)}{F(v^*)} = 1$; the inequality follows from Jensen's inequality; the last equality follows from the definition of $k(v^*)$ (see (20)). Thus, (29) implies

$$k(v^*) > \int_0^{v^*} v_i d\frac{F(v_i)}{F(v^*)}.$$

That is, the seller has more expected revenue in (σ_1^*, σ_2^*) than in $(\widehat{\sigma}_1, \widehat{\sigma}_2)$.$\blacksquare$

Proof of Theorem 2

First, since (σ_1^*, σ_2^*) is a PBE, for every i, we have

$$Eu_i\left(v_i|\sigma_i^*, \sigma_{-i}^*\right) \geqslant Eu_i\left(v_i|\widehat{\sigma}_i, \sigma_{-i}^*\right), \; \forall v_i \in [0,1]. \tag{11}$$

Furthermore, $\left(\widehat{\sigma}_i, \sigma_{-i}^*\right)$ and $(\widehat{\sigma}_i, \widehat{\sigma}_{-i})$ induce different outcomes if and only if $v_i < v^* \leqslant v_{-i}$. In particular, in such a case, bidder i loses in both $\left(\widehat{\sigma}_i, \sigma_{-i}^*\right)$ and $(\widehat{\sigma}_1, \widehat{\sigma}_2)$, i.e., $u_i\left(v_i|\widehat{\sigma}_i, \sigma_{-i}^*\right) = u_i\left(v_i|\widehat{\sigma}_1, \widehat{\sigma}_2\right) = 0$ if $v_i < v^* \leqslant v_{-i}$. Hence,

$$Eu_i\left(v_i|\widehat{\sigma}_i, \sigma_{-i}^*\right) = Eu_i\left(v_i|\widehat{\sigma}_i, \widehat{\sigma}_{-i}\right) \; \forall v_i \in [0,1]. \tag{12}$$

(11) and (12) imply $Eu_i\left(v_i|\sigma_i^*, \sigma_{-i}^*\right) \geqslant Eu_i\left(v_i|\widehat{\sigma}_i, \widehat{\sigma}_{-i}\right)$ for every $v_i \in [0,1]$.$\blacksquare$

C Experimental Instructions: *Baseline*

INSTRUCTION

Welcome to the experiment. This experiment studies decision making between two individuals. In the following hour or so, you will participate in 10 rounds of decision making. Please read the instructions below carefully; the cash payment you will receive at the end of the experiment depends on how you make your decisions according to these instructions. Communication of any kinds with any other participants will not be allowed.

Your Group

There are 20 participants in today's session. Prior to the first round, 20 people are equally and anonymously divided into 2 classes. Your class will remain fixed throughout the experiment. In each round you will be matched with another participant in your class to form a group of two. Participants will be randomly rematched after each round to form new groups, and each participant in your class have an equal chance to be matched with you. You will not be told the identity of the participant you are matched with, nor will that participant be told your identity—even after the end of the experiment.

Your Decision in Each Round

In your group, there are two individuals, yourself and your opponent. In each round and for each individual, the computer randomly and independently selects **Your Value** from 1 to 60. Each integer number between 1 and 60 has equal chance to be selected. At the beginning of each round, you will be informed about your value. You will not be told the value of the participant you are matched with, nor will that participant be told your value.

In each round, you are endowed with 60 tokens and are asked to **make a bid** to win an auction that consists of the following two stages: Initial Bidding Stage (Stage 1) and Price Clock Stage (Stag 2).

Stage 1: Initial Bidding Stage

You will be informed about your value and be asked to place your **Initial Bid** (see Figure 8). The initial bid can be any integer number between 0 and 60, inclusively. Once you input your initial bid, you click the submit button. Note that the **maximum** of your initial bid and your opponent's initial bid will become the **Initial Price** in the next stage, which will be explained further below.

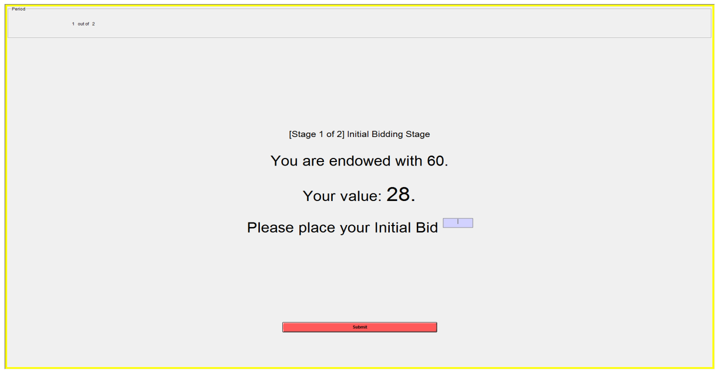
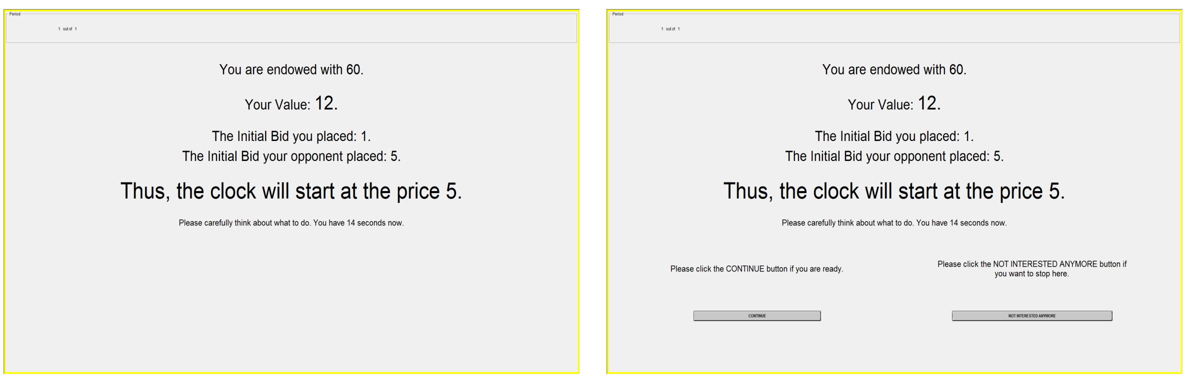


Figure 8: Stage 1 – Initial Bidding Stage

After you and your opponent click the submit button, you will be informed about the initial price as you see in Figure 9(a). You will be asked to think for a number of seconds on what to do next. The number of seconds you stay with the screen will be randomly determined between 5 seconds and 15 seconds. The waiting time is also independent upon your initial bids.



(a) Waiting Screen

(b) Continue or Opt-out

Figure 9: Initial Price and Opt-out Decision

If you submit an initial bid strictly lower than your opponent's initial bid (which is equal to the initial price), you will be asked to decide whether to **continue** (by clicking the CONTINUE button) or to **opt out** (by clicking the NOT INTERESTED ANYMORE button). (See Figure 9(b) for the details). If you opt out, your opponent wins the auction with the initial price; if you continue, you will proceed to Stage 2.

If you submit an initial bid higher than or equal to your opponent's initial bid, you will be asked to click the CONTINUE button to proceed to Stage 2.

Stage 2: Price Clock Stage

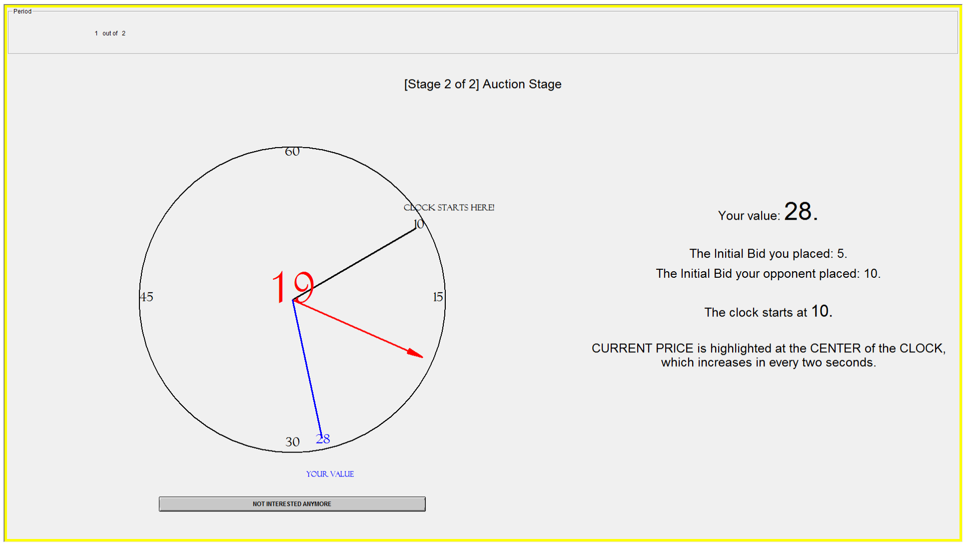


Figure 10: Stage 2 – Price Clock Stage

Figure 10 demonstrates an example of your decision screen in Stage 2. On the left-hand side of your screen, a **Price Clock** will be presented with three pieces of information on it: (1) Initial Price, (2) Current Price, and (3) Your Value.

(1) **Initial Price**: The price clock starts with the Initial Price determined in the Initial Bid Stage (= the maximum of the initial bids).

(2) **Current Price**: the current price is displayed at the center of the Clock (highlighted in **Red** colour). In every **two** seconds, the clock goes and the current price increases in 1 unit.

(3) **Your Value**: Your value is highlighted in the clock with **Blue** colour.

Under the price clock, a button "NOT INTERESTED ANYMORE" is in place. Whenever one of the individuals in your group clicks the button, the price clock stops and the auction ends. The individual who stays in the auction is declared the winner, and pays the price showing on the clock. Once an individual has opted out, he/she cannot re-enter. If no one drops out until the current price becomes 60, the auction ends and the final price becomes 60. In this case, each individual has equal chance to win the auction.

Your Earning in Each Round

- If you do not win the auction, your earning becomes your endowment 60.
- If you win the auction, your earning becomes

$$\text{Endowment} + \text{Your Value} - \text{Final Price}.$$

For example, when you win the auction with your value 28 and the final price 19, your earning in the round becomes $60 + 28 - 19 = 69$.

Information Feedback

At the end of each round, you will be informed about Your Value, Your Opponent's Value, Your Initial Bid, Your Opponent's Initial Bid, Final Price, Auction Outcome (win or lose) and Your Earning.

Your Cash Payment

The experimenter *randomly* selects 1 round to calculate your cash payment. (So it is in your best interest to take each round seriously). Your total cash payment at the end of the experiment will be the number of tokens you earned in the selected round (translated into HKD with the exchange rate of 1 Token = 1 HKD) plus a 30 HKD show-up fee.

Practice Rounds

To ensure your comprehension of the instructions, we will provide you with a practice round. Once the practice round is over, the computer will tell you "The official rounds begin now!"

Administration

Your decisions as well as your monetary payment will be kept confidential. Remember that you have to make your decisions entirely on your own; please do not discuss your decisions with any other participants. Upon finishing the experiment, you will receive your cash payment. You will be asked to sign your name to acknowledge your receipt of the payment. You are then free to leave. If you have any question, please raise your hand now. We will answer your question individually.

D Instructions for Eliciting Risk Attitudes

INSTRUCTION- Bonus I

Please read the instructions carefully and make decisions. In the table below, there are 12 decisions to be made. Each row presents each decision. In each row, you need to choose one of two options, Option A and Option B.

- If you choose Option A, you will get either HKD 14 (Outcome A1) or HKD 6 (Outcome A2) depending on the realization of X on the Orange Card.
- X will be randomly drawn in the range between [1,100] inclusively. Each integer number in this range has an equal chance to be selected.
- If you choose Option B, you will get HKD 10 regardless of the realization of X on the Orange Card.
- Please make your decisions for all 12 rows and click SUBMIT / OPEN THE CARD button. Then, one row will be randomly selected and the selected row number will be presented on the Green Card. Each row has equal chance to be selected.
- Your earning in this bonus round will be determined by your decision for the selected row and the realization of X.

Please raise your hand if you have any questions.
Otherwise, please make your decision.

INSTRUCTION- Bonus II

In the table below, there are twelve decisions to be made. Each row presents each decision. Everything is the same as before except the followings:

- If you choose Option A, you will get either HKD 34 (Outcome A1) or HKD 26 (Outcome A2) depending on the realization of X on the Orange Card.
- If you choose Option B, you will get HKD 30 regardless of the realization of X on the Orange Card.

Please raise your hand if you have any questions.
Otherwise, please make your decision.

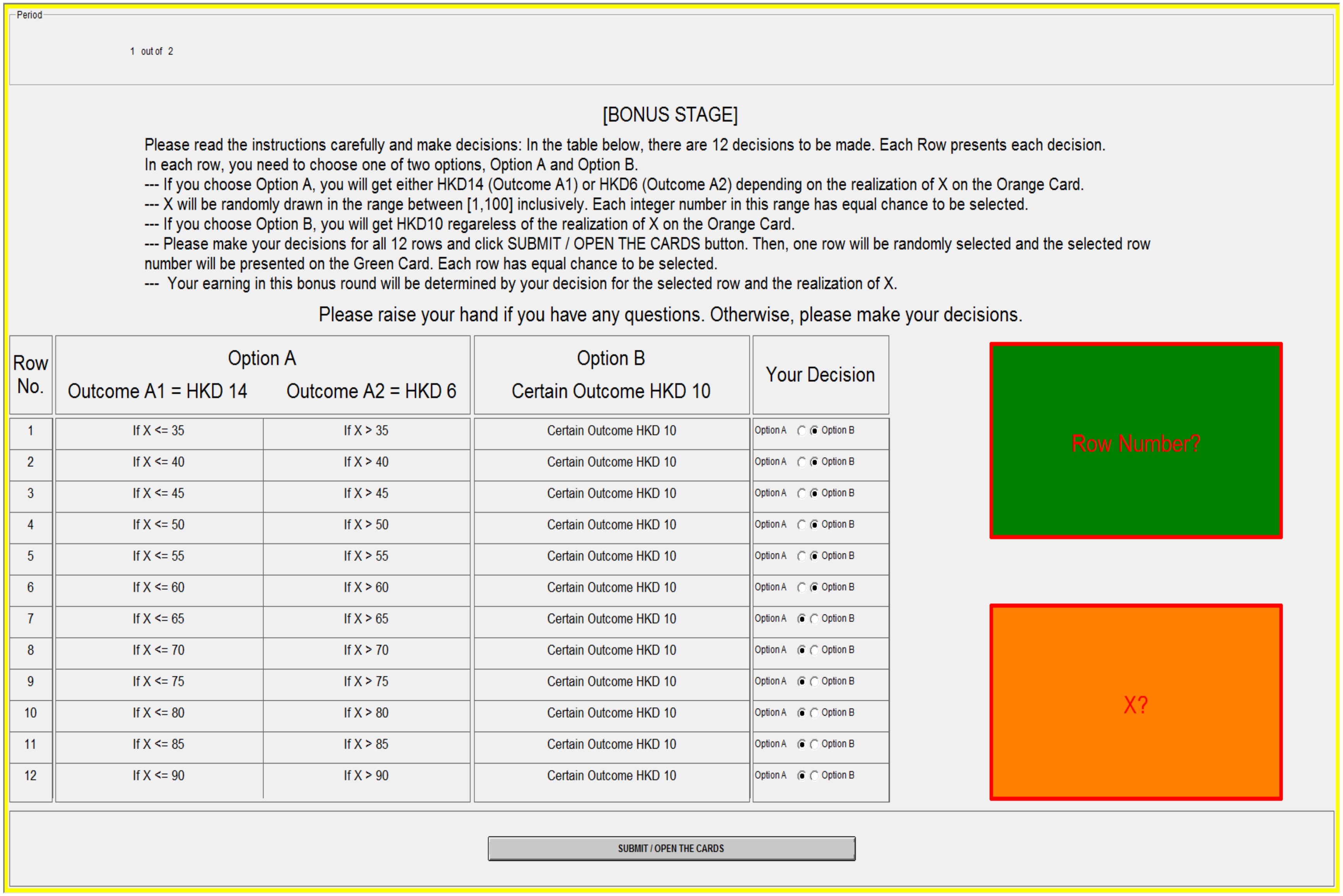


Figure 11: Z-tree Screen Shot - Bonus I

E Online Appendix: Equilibria with Two Rounds of Jump

We now define a class of 2-round-jump-bidding equilibria. In particular, we define below $([v^*, k^*(v^*)], [v^{**}, k^{**}(v^{**})])$, where, in the first round, a bidder jumps to $k^*(v^*)$ if and only if her value is weakly higher than v^*, and in the second round, a bidder jumps to $k^{**}(v^{**})$ if and only if her value is weakly higher than v^{**}.

We modify the model to accommodate 2-round jump bidding as follows. There is a jump stage (i.e., stage 1), followed by a standard clock auction (i.e., stage 2). In stage 1, each bidder i chooses $\left(\beta_i^1, \beta_i^2\right) \in [0,1] \times \left([0,1]^{[0,1]\times[0,1]}\right)$ such that

$$\beta_i^2\left(\beta_1^1, \beta_2^1\right) \geqslant \max\left\{\beta_1^1, \beta_2^1\right\},$$

i.e., in the first round of jump bidding, bidder i jumps to β_i^1, and upon observing $\left(\beta_1^1, \beta_2^1\right)$, bidder i jumps to $\beta_i^2\left(\beta_1^1, \beta_2^1\right)$ in the second round of jump bidding. For notational ease, we use β_i^2 to denote $\beta_i^2\left(\beta_1^1, \beta_2^1\right)$, with the understanding that β_i^2 is the i's 2nd-round jump price (upon observing $\left(\beta_1^1, \beta_2^1\right)$).

In stage 2, an English auction with the starting price $\max\left\{\beta_1^2, \beta_2^2\right\}$ is conducted, and each bidder i, upon observing the jump prices (i.e., $\left[\left(\beta_1^1, \beta_2^1\right), \left(\beta_1^2, \beta_2^2\right)\right]$), chooses the price $b_i\left(\left[\left(\beta_1^1, \beta_2^1\right), \left(\beta_1^2, \beta_2^2\right)\right], v_i\right) \geqslant \max\left\{\beta_1^2, \beta_2^2\right\}$ to exit. For notational ease, we use b_i to denote $b_i\left(\left[\left(\beta_1^1, \beta_2^1\right), \left(\beta_1^2, \beta_2^2\right)\right], v_i\right)$.

A winner gets utility $u\left[v_i - b_{-i}\right]$, and his opponent (i.e., the loser) gets 0. Bidder i wins if $b_i > b_{-i}$, and a tie occurs if and only if $b_i = b_{-i}$. Whenever a tie occurs, we use a fair coin to determine the winner.

Throughout the paper, we adopt the solution concept of perfect Bayesian equilibrium (PBE). As a benchmark, the usual no-jump equilibrium is defined as follows.

$$\widetilde{\sigma}_i : \begin{pmatrix} \text{stage 1: } \beta_i^1 = 0 \text{ and } \beta_i^2 \equiv \max\left\{\beta_1^1, \beta_2^1, v_i\right\}; \\ \text{stage 2: } b_i = \max\left\{\beta_1^2, \beta_2^2, v_i\right\}. \end{pmatrix}$$

It is straightforward to see that $(\widetilde{\sigma}_i, \widetilde{\sigma}_{-i})$ is an equilibrium.

For any $v \in [0,1]$, define $k^*(v) \in [0,1]$ to be the unique number satisfying[41]

$$u(v - k^*(v)) = \int_{v' \in [0,v]} u(v - v') d\frac{F(v')}{F(v)}. \tag{13}$$

Fix any $v^* \in (0,1)$ and the corresponding $k^*(v^*)$ as determined by (13). I.e., v^* is bidders' threshold value to jump and $k^*(v)$ is the jump price. Suppose that both bidders' values are weakly higher than v^*, and we proceed to the second round of jump bidding. For any $v \in [v^*,1]$, define $k^{**}(v) \in [0,1]$ to be the unique number satisfying[42]

$$u(v - k^{**}(v)) = \int_{v' \in [v^*,v]} u(v - v') d\frac{F(v') - F(v^*)}{F(v) - F(v^*)}. \tag{14}$$

Fix any $v^{**} \in (0,1)$ and the corresponding $k^{**}(v^{**})$ as determined by (14). I.e., v^{**} is bidders' threshold value to jump and $k^{**}(v^{**})$ is the jump price.

Given $([v^*, k^*(v^*)], [v^{**}, k^{**}(v^{**})])$ fixed abvoe, we construct a 2-round-jump-bidding equilibrium $(\sigma_1^{**}, \sigma_2^{**})$ as follows.

$$\sigma_i^{**}: \left(\begin{array}{l} \text{stage 1:} \left(\begin{array}{c} \beta_i^1 = \begin{cases} k^*(v^*), & \text{if } v_i \geqslant v^*; \\ 0, & \text{if } v_i < v^*. \end{cases} \\ \\ \beta_i^2\left(\beta_i^1, \beta_{-i}^1\right) = \begin{cases} k^{**}(v^{**}), & \begin{array}{l} \text{if } \left[\beta_i^1, \beta_{-i}^1\right] = [k^*(v^*), k^*(v^*)] \\ \text{and } v_i \geqslant v^{**}; \end{array} \\ \\ \max\left\{\beta_i^1, \beta_{-i}^1, v_i\right\}, & \text{otherwise.} \end{cases} \end{array} \right) \\ \\ \text{stage 2: } b_i\left(\beta_i, \beta_{-i}, v_i\right) = \begin{cases} \max\left\{\beta_i^2,\ \beta_{-i}^2\right\}, & \begin{array}{l} \text{if } \left[\beta_i^1, \beta_{-i}^1\right] = [k^*(v^*), k^*(v^*)] \\ \text{and } \beta_{-i}^2 = k^{**}(v^{**}) \\ \text{and } v_i < v^{**}; \end{array} \\ \\ \max\left\{\beta_i^2,\ \beta_{-i}^2, v_i\right\}, & \text{otherwise.} \end{cases} \end{array} \right)$$

[41] $u(v-y)$ is strictly decreasing in y. Since $u(v-0) \geqslant \int_{v' \in [0,v]} u(v-v') d\frac{F(v')}{F(v)} \geqslant u(v-v)$, there exists a unique $k^*(v)$ for each $v \in [0,1]$ such that equation (13) is satisfied.

[42] $u(v-y)$ is strictly decreasing in y. Since $u(v-v^*) \geqslant \int_{v' \in [v^*,v]} u(v-v') d\frac{F(v')-F(v^*)}{F(v)-F(v^*)} \geqslant u(v-v)$, there exists a unique $k^{**}(v)$ for each $v \in [v^*,1]$ such that equation (14) is satisfied.

With the same argument as in Lim and Xiong (2020), it is straightforward to show the following results.

Proposition 3. $(\sigma_1^{**}, \sigma_2^{**})$ *is a PBE for risk-neutral, CARA and IARA bidders.*

Theorem 3 (seller's revenue)**.** *Given risk-averse bidders, the seller has more expected revenue in* $(\sigma_1^{**}, \sigma_2^{**})$ *than in* $(\widetilde{\sigma}_1, \widetilde{\sigma}_2)$*.*

F Online Appendix: Jump-bidding with Bidding Cost

In this note, we show how bidding cost (e.g., impatience) can be embedded into the jump-bidding model in Lim and Xiong [2].

F.1 The Model

For simplicity, we consider a 2-bidder independent private value (hereafter, IPV) model. One indivisible object is for sale, with bidders 1 and 2 having values v_1, v_2, respectively. The values have i.i.d. distribution on the support $[0, 1]$ with cdf $F(\cdot)$. The two bidders are expected utility maximizers with the same differentiable and strictly increasing Bernoulli utility function $u(\cdot)$. We normalize $u(0)$ to 0. Let bidder $-i$ denote bidder i's opponent.

We model the auction by a 2-stage game: the jump stage (i.e., stage 1), followed by the bidding stage (i.e., stage 2). In stage 1, each bidder i chooses $\beta_i \in [0, 1]$; in stage 2, a standard English auction with the starting price $\max\{\beta_i, \beta_{-i}\}$ is conducted, and each bidder i chooses the price $b_i(\beta_i, \beta_{-i}, v_i) \geqslant \max\{\beta_i, \beta_{-i}\}$ to exit. For instance, "$b_{-i}(\beta_i, \beta_{-i}, v_{-i}) = \beta_i > \beta_{-i}$" means that bidder $-i$ quits immediately after bidder i jumps to β_i. Bidder i wins the auction if and only if $b_i(\beta_i, \beta_{-i}, v_i) \geqslant b_{-i}(\beta_i, \beta_{-i}, v_{-i})$." If $b_i(\beta_i, \beta_{-i}, v_i) = b_{-i}(\beta_i, \beta_{-i}, v_{-i})$, and let us fix any tie-breaking rule.

We say bidder i enters the auction if and only if $\beta_i > 0$ or $b_i(\beta_i, \beta_{-i}, v_i) > \max\{\beta_i, \beta_{-i}\}$. Entering the auction is costly: a bidder incurs an additional cost of $c > 0$ if only if he enters the auction. Here, c models cognitive cost and/or psychological cost (e.g., impatience). For example, if $b_i(\beta_i, \beta_{-i}, v_i) > b_{-i}(\beta_i, \beta_{-i}, v_{-i}) > \max\{\beta_i, \beta_{-i}\}$, bidder i wins and his utility is $u[v_i - b_{-i}(\beta_i, \beta_{-i}, v_{-i})] - c$, and bidder $-i$ loses and his utility is $-c$.

Throughout the paper, we adopt the solution concept of perfect Bayesian equilibrium (PBE).

F.2 The No-Jump Equilibrium

Define e^* to be unique number satisfying

$$F(e^*) \times u(e^*) - c = 0, \tag{15}$$

Define e^{**} to be the unique number satisfying

$$u(e^{**}) - c = 0. \tag{16}$$

Note that $e^{**} < e^*$.

For simplicity, we assume bidders' values follow the i.i.d. uniform distribution on $[0,1]$, i.e.,

$$F(v) = v \text{ and } f(v) = 1,\ \forall v \in [0,1].$$

Moreover, we assume that c is sufficiently small so that

$$u'(v) > u(e^*),\ \forall v \in [0,1]. \tag{17}$$

The usual no-jump equilibrium has to be modified slightly. For instance, it is a strictly dominant strategy for a type with a sufficiently low value (i.e., $v_i < e^{**}$) not to enter the auction.

$$\widehat{\sigma}_i : \left(\begin{array}{c} \text{stage 1: } \beta_i = 0; \\ \text{stage 2: } b_i(\beta_i, \beta_{-i}, v_i) = \begin{cases} 0, & \text{if } \beta_i = \beta_{-i} = 0 \text{ and } v_i < e^*; \\ \max\{\beta_i, \beta_{-i}\}, & \text{if } \beta_i = 0, \beta_{-i} > 0 \text{ and } v_i < \beta_{-i} + e^{**}; \\ \max\{\beta_i, \beta_{-i}, v_i\}, & \text{otherwise.} \end{cases} \end{array} \right)$$

Proposition 4 (no-jump PBE). *Given risk-averse bidders, $(\widehat{\sigma}_1, \widehat{\sigma}_2)$ is a PBE.*

Proof of Proposition 4 First, on the equilibrium path, no one jumps in stage 1, and each bidder i enters the English auction in stage 2 if and only if $v_i \geqslant e^*$. Indeed, by (15), given this strategy, each bidder i finds it (weakly) profitable to enter the auction if and only if $v_i \geqslant e^*$.

Second, once bidder i enters the auction, the bidding cost is a sunk cost, and $\max\{\beta_i, \beta_{-i}, v_i\}$ is the weakly dominant strategy in the English auction in stage 2.

Third, consider any off-equilibrium jump (in stage 1) faced by bidder i (i.e., $\beta_{-i} > 0$). Suppose bidder i believes $v_{-i} = \beta_{-i}$, and bidder $-i$ would quit immediately in stage 2. (16) implies that bidder i finds it (weakly) profitable to enter the auction if and only if $v_i \geqslant \beta_{-i} + e^{**}$.

Finally, we show bidder i does not find it profitable to deviate to any off-equilibrium jump in stage 1. Suppose type v_i is considering jumping to $\beta_i > 0$ in stage 1. It is without loss of generality to assume

$$e^* \leqslant \beta_i + e^{**} \leqslant v_i. \tag{18}$$

If $v_i < \beta_i + e^{**}$, the maximal utility for jumping to β_i is $u(v_i - \beta_i) - c < u(e^{**}) - c = 0$. Also, if $\beta_i + e^{**} \leqslant e^*$, bidder i has to pay a higher price (i.e., $\beta_i > 0$) to drive out a smaller proportion of opponents (i.e., $v_{-i} \leqslant \beta_i + e^{**}$). In contrast, if bidder i follows $\widehat{\sigma}_i$, he pays 0 to drive out a bigger proportion of opponents (i.e., $v_{-i} \leqslant e^*$). Therefore, jumping to β_i with $\beta_i + e^{**} \leqslant e^*$ is strictly worse than $\widehat{\sigma}_i$.

Moreover, for risk-averse bidders, $u(\cdot)$ is concave. Then, $\beta_i + e^{**} \leqslant v_i$ implies

$$u(v_i - \beta_i) - u(v_i - \beta_i - e^{**}) \leqslant u(\beta_i + e^{**} - \beta_i) - u(\beta_i + e^{**} - \beta_i - e^{**}) = u(e^{**}) = c, \tag{19}$$

where, the first inequality follows concavity of $u(\cdot)$, the first equality from $u(0) = 0$, and the last equality from (16).

Define

$$\varsigma(\beta_i) := F(\beta_i + e^{**}) \times u(v_i - \beta_i) + \int_{\beta_i + e^{**}}^{v_i} u(v_i - v_{-i})\, dF(v_{-i}) - c,$$

and $\varsigma(\beta_i)$ is the expected utility for jumping to β_i in stage 1. Then, by the uniform distribution, we have

$$\begin{aligned}
\varsigma'(\beta_i) &= [u(v_i-\beta_i)-u(v_i-\beta_i-e^{**})]-F(\beta_i+e^{**})\times u'(v_i-\beta_i) \\
&\leqslant c-F(\beta_i+e^{**})\times u'(v_i-\beta_i) \\
&\leqslant c-F(e^*)\times u'(v_i-\beta_i) \\
&< c-F(e^*)\times u(e^*)=0,
\end{aligned}$$

where the first inequality follows from (19); the second inequality follows from (18); the third inequality follows from (17); the last equality follows from (15).

That is, given $e^* \leqslant \beta_i + e^{**} \leqslant v_i$, if type v_i has to make an off-equilibrium jump, the best jump price is $e^* - e^{**}$, i.e., $\beta_i + e^{**} = e^*$. As argued above, jumping to this price is strictly worse than $\widehat{\sigma}_i$.■

F.3 The Jump-Bidding Equilibrium

Fix any $v^* \in (e^*, 1)$. Define $k(v^*) \in [0,1]$ as the unique number satisfying

$$F(v^*)\times u[v^*-k(v^*)] = F(e^*)\times u(v^*) + \int_{e^*}^{v^*} u(v^*-v)dF(v). \tag{20}$$

We will construct a jump-bidding equilibrium (σ_1^*, σ_2^*), in which v^* is the threshold value for the bidders to jump bid in stage 1 and $k(v^*)$ is the jump price. That is, bidder i jumps to $k(v^*)$ in stage 1 if and only if $v_i \geqslant v^*$.

$$\sigma_i^* : \left(\begin{array}{c}
\text{stage 1: } \beta_i = \begin{cases} 0, & \text{if } v_i < v^*; \\ k(v^*), & \text{if } v_i \geqslant v^*. \end{cases} \\
\text{stage 2: } b_i(\beta_i, \beta_{-i}, v_i) = \begin{cases} 0, & \text{if } \beta_i = \beta_{-i} = 0 \text{ and } v_i < e^*; \\ \max\{\beta_i, \beta_{-i}\}, & \text{if } \beta_i = 0,\, \beta_{-i} = k(v^*) \text{ and } v_i < v^*; \\ \max\{\beta_i, \beta_{-i}\}, & \begin{array}{c}\text{if } \beta_i = 0,\ \beta_{-i} \notin \{0, k(v^*)\} \\ \text{and } v_i < \beta_{-i} + e^{**};\end{array} \\ \max\{\beta_i, \beta_{-i}, v_i\}, & \text{otherwise.} \end{cases}
\end{array} \right)$$

In stage 1, bidder i uses the jump price $k(v^*)$ to signal his high value, i.e., $v_i \geqslant v^*$.

Suppose bidder i does not jump in stage 1, i.e., $\beta_i = 0$. Then, bidder i does not enter the auction in stage 2 in three cases: i) his value is too low, i.e., $v_i < e^*$; ii) $\beta_{-i} = k(v^*)$ and $v_i < v^*$, i.e., $\beta_{-i} = k(v^*)$ implies his opponent has high value (i.e., $v_{-i} \geqslant v^*$), and the low types of bidder i (i.e., $v_i < v^*$) expect no chance to win; iii) $\beta_{-i} \notin \{0, k(v^*)\}$ and $v_i < \beta_{-i} + e^{**}$, i.e., for any off-equilibrium jump β_{-i}, type $v_i < \beta_{-i} + e^{**}$ does not find it profitable to enter the auction.

By the same argument in the proof of Proposition 1, bidder i does not find it profitable to deviate to any off-equilibrium jump in stage 1, given bidder $-i$ taking σ^*_{-i}. Hence, (σ^*_1, σ^*_2) is a PBE if and only if the bidders' signalling in stage 1 is credible, i.e., the high types prefer "jumping to $k(v^*)$" and the low types prefer "no jump." Consider type $v_i \geqslant e^*$, define

$$N\left(v_i\right) := F\left(e^*\right) \times u\left(v_i\right) + \int_{e^*}^{v_i} u(v_i - v_{-i}) dF(v_{-i}) - c. \tag{21}$$

$$J\left(v_i\right) := F\left(v^*\right) \times u(v_i - k(v^*)) + \int_{v^*}^{\max\{v_i,\, v^*\}} u(v_i - v_{-i}) dF(v_{-i}) - c. \tag{22}$$

$N\left(v_i\right)$ and $J\left(v_i\right)$ are the expected utility for "no jump" and "jumping to $k(v^*)$," respectively. Note that $N\left(v^*\right) = J\left(v^*\right)$, i.e., $k(v^*)$ in (20) is chosen so that the threshold type v^* is indifferent between no jump and jumping to $k(v^*)$.

Lemma 2. *Given risk-averse bidders, (σ^*_1, σ^*_2) is a PBE if and only if*

$$N\left(v\right) \geqslant J\left(v\right) \text{ for every } v \in [e^*,\ v^*) \text{ and } N\left(v\right) \leqslant J\left(v\right) \text{ for every } v \in [v^*,\ 1].$$

Proposition 5. *Given risk-averse bidders, (σ^*_1, σ^*_2) is a PBE for CARA and IARA bidders.*

Proof of Proposition 5. Define a new cdf as follows.

$$\widetilde{F}\left(v_{-i}\right) = \begin{cases} \frac{F(e^*)}{F(v^*)}, & \text{if } v_{-i} \in [0, e^*]\,; \\ \frac{F(v_{-i})}{F(v^*)}, & \text{if } v_{-i} \in (e^*, v^*]. \end{cases} \tag{23}$$

By (20), we have

$$u\left[v^* - k(v^*)\right] = \int_0^{v^*} u(v^* - v_{-i}) d\widetilde{F}\left(v_{-i}\right). \tag{24}$$

Define

$$g\left(v_i\right) := u(v_i - k(v^*));$$

$$h\left(v_i\right) := E_{v_{-i}\sim[0,v^*]\text{ with cdf }\widetilde{F}(\cdot)}\left[u(v_i - v_{-i})\right] = \int_0^{v^*} u(v_i - v_{-i})d\widetilde{F}\left(v_{-i}\right)$$
$$= \frac{F\left(e^*\right)}{F(v^*)} \times u\left(v_i\right) + \int_{e^*}^{v_i} u(v_i - v_{-i})d\frac{F(v_{-i})}{F(v^*)}.$$

By (21) and (22), we have

$$J\left(v_i\right) - N\left(v_i\right) = \begin{cases} F(v^*) \times \left[g\left(v_i\right) - h\left(v_i\right)\right] - \int_{v^*}^{v_i} u(v_i - v_{-i})dF(v_{-i}), & \text{if } v_i \in [e^*, v^*); \\ F(v^*) \times \left[g\left(v_i\right) - h\left(v_i\right)\right], & \text{if } v_i \in [v^*, 1], \end{cases}$$

which implies

$$J\left(v_i\right) - N\left(v_i\right) \begin{cases} \leqslant F(v^*) \times \left[g\left(v_i\right) - h\left(v_i\right)\right], & \text{if } v_i \in [e^*, v^*); \\ = F(v^*) \times \left[g\left(v_i\right) - h\left(v_i\right)\right], & \text{if } v_i \in [v^*, 1], \end{cases} \tag{25}$$

because $\int_{v^*}^{v_i} u(v_i - v_{-i})dF(v_{-i}) \geqslant 0$ for every $v_i < v^*$.

The properties of IARA and CARA (see Matthews [1], p.638) imply

$$IARA: \; g\left(v_i\right) = h\left(v_i\right) \implies g'\left(v_i\right) > h'\left(v_i\right); \tag{26}$$

$$CARA: \; g\left(v_i\right) = h\left(v_i\right) \implies g'\left(v_i\right) = h'\left(v_i\right). \tag{27}$$

Note that (26) implies $g(\cdot)$ and $h(\cdot)$ cross at most once. In particular, $g(v^*) = h(v^*)$. Hence,

$$IARA: g\left(v_i\right) - h\left(v_i\right) \begin{cases} < 0, & \text{if } v_i < v^*; \\ = 0, & \text{if } v_i = v^*; \\ > 0, & \text{if } v_i > v^*. \end{cases} \text{ and } CARA: g(v_i) - h(v_i) = 0, \forall v_i \in [e^*, 1]. \tag{28}$$

(25) and (28) imply

$$IARA \,\&\, CARA: J\left(v_i\right) - N\left(v_i\right) \begin{cases} \leqslant 0 & \text{if } v_i \in [e^*, v^*); \\ \geqslant 0 & \text{if } v_i \in [v^*, 1]. \end{cases}$$

Therefore, (σ_1^*, σ_2^*) is a PBE by Lemma 2.■

Finally, the revenue effect remains.

Theorem 4. *Given risk-averse bidders, the seller has more expected revenue in* (σ_1^*, σ_2^*) *than in* $(\widehat{\sigma}_1, \widehat{\sigma}_2)$.

Proof of Theorem 4. There are three events: i) $[\max\{v_1, v_2\} < v^*]$, ii) $[v^* \leqslant \min\{v_1, v_2\}]$ and iii) $[\min\{v_1, v_2\} < v^* \leqslant \max\{v_1, v_2\}]$. In event of i) or ii), the seller gets the same revenue in both (σ_1^*, σ_2^*) and $(\widehat{\sigma}_1, \widehat{\sigma}_2)$.

Conditional on event iii) $[\min\{v_1, v_2\} < v^* \leqslant \max\{v_1, v_2\}]$, the expected revenues in (σ_1^*, σ_2^*) and $(\widehat{\sigma}_1, \widehat{\sigma}_2)$ are $k(v^*)$ and $\int_0^{v^*} v d\widetilde{F}(v)$, respectively, where $\widetilde{F}(\cdot)$ is defined in (23).

Given risk-averse bidders, $u(\cdot)$ is strictly concave and

$$u\left[v^* - \int_0^{v^*} v d\widetilde{F}(v)\right] = u\left[\int_0^{v^*} (v^* - v)\, d\widetilde{F}(v)\right] > \int_0^{v^*} u(v^* - v)\, d\widetilde{F}(v) = u(v^* - k(v^*)), \tag{29}$$

where the first equality follows from $\int_0^{v^*} d\widetilde{F}(v) = 1$; the inequality follows from Jensen's inequality; the last equality follows from (24). Thus, (29) implies

$$k(v^*) > \int_0^{v^*} v d\widetilde{F}(v).$$

That is, the seller has more expected revenue in (σ_1^*, σ_2^*) than in $(\widehat{\sigma}_1, \widehat{\sigma}_2)$.■

References

[1] S. Matthews, Comparing auctions for risk averse buyers: a buyer's point of view, Econometrica, 55 (1987), 633-646.

[2] Lim W. and S. Xiong, Does Jump-bidding Increase Sellers' Revenue? Theory and Experiment, Mimeo, 2020.