

Voluntary Participation in Public Goods Provision with Coasian Bargaining*

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Abstract

This paper reports findings from an experimental study of voluntary participation games, as considered by Dixit and Olson [“Does voluntary participation undermine the Coase Theorem?” *Journal of Public Economics*, (2000), 76: 309-335]. The voluntary participation game consists of two stages: a non-cooperative participation decision followed by Coasian bargaining on public goods provision only among those who choose to participate. Our experimental findings show that, consistent with the theoretical findings of Dixit and Olson, the outcome of this game falls short of full efficiency. However, we find that voluntary participation undermines the Coase Theorem to a lesser extent than predicted by Dixit and Olson, particularly with larger numbers of players. We also investigate the effect of pre-play communication on the public goods provision and find little evidence that cheap talk helps subjects coordinate on the efficient outcome of coalition formation in the laboratory.

Keywords: Coase Theorem, Voluntary Participation, Public Goods, Laboratory Experiments

JEL classification: C91; D71; D83; H41

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1 Introduction

Does voluntary participation undermine the efficient provision of public goods? In this paper, we investigate voluntary participation in a public goods provision game and report findings from a series of laboratory experiments. The international arena of climate negotiation showcases the importance of voluntary participation in public goods provision. In the absence of an international government, the world has to rely on countries' voluntary participation in climate treaties to resolve the climate problem. For instance, the Kyoto Protocol may be an effective mechanism to provide a global public good - the abatement in greenhouse gas emissions - if all of the main emitters opt to commit to it. To successfully enact the Protocol, two conditions must be met: 1) at least 55 parties must ratify it and 2) those parties that are listed in Annex I of the Protocol and have ratified it must account for at least 55% of the total carbon dioxide emissions at the 1990 level for the Annex I countries. The "55 parties" clause was satisfied when Iceland ratified the Protocol, and the ratification by Russia satisfied the "55%" clause. The United States, the largest emitter of greenhouse gases in Annex I, however, did not ratify the Kyoto Protocol. Without the participation of the United States, the climate treaty had little chance of successfully addressing the climate problem.

Several recent papers conduct theoretical investigations into voluntary participation in public goods provision. Saijo and Yamato (1999) consider a two-stage game in which a first stage voluntary participation is followed by a mechanism in the second stage satisfying several desirable properties such as non-emptiness, feasibility, symmetry, and Pareto efficiency for participants. They show that the most efficient outcome with the participation of all agents is often not supported by an equilibrium. Saijo and Yamato (2010) extend the model and show an impossibility theorem of full participation. In their influential paper, Dixit and Olson (2000) look at the mixed strategy participation equilibrium of a similar game with a lumpy public good.¹ In the second stage of their model, participants engage in Coasian Bargaining to determine whether or not to provide the public good. Dixit and Olson primarily rely on simulations to support the claim that voluntary participation undermines the Coase Theorem (Coase, 1960): public goods are unlikely to be provided if the provision of public goods relies solely on the voluntary participation choices of individuals. Shinohara (2009) uses a similar model to demonstrate the existence of refined pure strategy Nash Equilibria with an efficient allocation.

The main purpose of this paper is to study experimentally the role of voluntary participation in a public good game. Although Saijo and Yamato (1999, 2010) mainly provide results on the impossibility of full participation, we aim to provide a quantitative insight on the partic-

¹A public good is said to be lumpy if no amount of the public good can be provided until the total contributions exceed some threshold (Taylor, 1987).

icipation level. In particular, we make heavy use of Dixit and Olson (2000).² Starting from the model considered by Dixit and Olson (2000), we first obtain some additional analytical results with new findings regarding the mixed strategy participation equilibrium. We then propose and conduct laboratory experiments to test the results from both Dixit and Olson (2000) as well as our own additional analysis, and investigate whether voluntary participation really undermines the Coase Theorem.

Our baseline treatment consists of two games with different group sizes ($N = 4$ and $N = 8$). This choice allows us to investigate how the group size affects participation. Controlling for the group size is of importance since Dixit and Olson (2000) are particularly pessimistic about the outcome when the group size is large. Olson (1965) argues that a public good would be provided more easily for a smaller group than for a larger one. Several papers (e.g., Harrington 2001, Heijnen 2009) follow up by theoretically examining the possibility that a failure in public goods provision occurs more often when the number of players involved increases. In reality, likewise, it was argued that a smaller group of countries might have a better chance of achieving a successful climate agreement, before the Copenhagen negotiations took place in 2009. However, the experimental economics literature on voluntary contribution mechanisms (VCM) provides different insights about the effect of group size on public goods provision. For instances, Isaac and Walker (1988a) find that allocative efficiency decreases in group size when marginal per capita return (MPCR) also declines, while there is no a pure numbers-in-the-group effect when MPCR is kept constant; Isaac, Walker and Williams (1994) find that larger groups may even provide public goods more efficiently. Different from these VCM games where contribution is a dominated strategy as in a prisoner's dilemma, participation is not a dominated strategy in our coalition game with Coasian bargaining, which is essentially a coordination game.

The experimental data from our baseline games show that the average frequency of participation decreases with the number of players, as predicted by Dixit and Olson (2000). Furthermore, consistent with the theoretical predictions, the reported outcomes of the baseline games fall short of full efficiency. However, we find that the voluntary participation undermines the Coase Theorem to a lesser extent than that predicted by Dixit and Olson (2000), particularly for the larger group size ($N = 8$). The “over-participation” of individual subjects is stronger in larger groups; the frequency of coalition formation (or equivalently, frequency of public goods provision) is significantly higher when $N = 8$ compared to the frequency of coalition formation when $N = 4$.

We demonstrate that risk-aversion increases the probability of the successful provision

²We focus on the one-shot version of the game in Dixit and Olson (2000). They also characterize infinitely repeated plays of the game, which lies beyond the interest of the current paper.

of public goods in the mixed strategy participation equilibrium characterized by Dixit and Olson (2000).³ Our finding is in line with several experimental studies showing that individuals are typically risk averse in the laboratory. Using the maximum likelihood method, we estimate the degree of risk-aversion in each treatment and find that a reasonable degree of risk-aversion is able to explain the pattern of “over-participation” observed in the lab. This result highlights that the pessimistic point of view by Dixit and Olson (2000) might be due to the fact that they assume risk-neutrality. Meanwhile, *conditional participation*, that is, subjects’ higher likelihood of participation if they believe more others will participate, seems to contribute to the higher frequency of coalition formation when $N = 8$.

We further study the effect of communication on the public goods provision in Dixit and Olson (2000).⁴ We introduce a communication stage prior to the participation stage in which each player sends costless and non-binding binary messages to everyone else in the group about his/her intention to participate in the coalition. Although participation is voluntary and a mechanism (a coalition) could be effective only for its members, there seems to be no reason to assume that players cannot communicate before they form a coalition. As noted by Crawford (1990), nothing can prevent bargainers from communicating by sending non-binding messages with no direct payoff implications.

Our theoretical analysis shows that introducing pre-play communication expands the set of equilibria of the game. More importantly, there exists an equilibrium in which cheap-talk messages sometimes allow players to successfully coordinate on the public goods provision; as a result, the *ex-ante* probability of coalition formation is strictly higher than that from the mixed-strategy participation equilibrium of the game without communication. Our communication treatment, which consists of two games ($N = 4$ and $N = 8$), is designed to test this prediction from our theoretical analysis. Our experimental finding shows that although communication sometimes affects players’ behavior and even improves coordination, the overall

³In contrast to this result, Teyssier (2012) shows that risk-aversion reduces contributions in a public good game with strategic uncertainty. The strategic uncertainty in Teyssier (2012) comes from the fact that first movers are unaware of second movers’ actions in sequential contributions. In Dixit and Olson (2000), strategic uncertainty is created by mixed strategies of simultaneous participation decisions.

⁴Examining the effects of communication in strategic interactions is of general interest in economics (see Crawford, 1998). A number of experimental studies investigate the role of communication in public goods provision. Isaac and Walker (1988b) present experimental evidence that the ability to talk among group members participating in a public goods game leads to increased cooperation in the form of higher contributions and lower free-riding. Palfrey and Rosenthal (1991) investigate a public goods game in which player endowments are private information, and find mixed evidence for cheap talk improving coordination. Chaudhuri et al. (2006) consider a novel type of communication by allowing players to pass advice to “later generations”. Orbell, Dawes and van de Kragt (1990) argue that communication “works either because it provides an occasion for (multilateral) promises or because it generates group identity.” See Ledyard (1995) and Chaudhuri (2011) for excellent surveys of the related literature and Zelmer (2003) for a meta-analysis. We are particularly interested in how communication affects participation decisions and coalition formation. Bolton and Chatterjee (1996) introduce communication in an experiment on coalition formation but not in the context of public goods provision.

effects of communication on participation and coalition formation are not statistically significant. A similar finding was previously documented by Bochet, Page and Putterman (2006), who showed that verbal communication (i.e., face-to-face or chat room) strongly increased cooperation, while numerical messages affected neither efficiency nor contributions.

Our experimental investigation belongs to an emerging line of experimental literature that considers public goods provision using a participation (coalition) game. Cason, Saijo and Yamato (2002) and Cason et al. (2004) let subjects decide whether to participate in public good contribution but instead of forming a coalition those who choose to participate then make their individual contribution decisions; they highlight the role of spiteful behavior in public good contribution. More recently, Dannenberg, Lange and Sturm (2010) and McEvoy and Cherry (2010), in contrast to Dixit and Olson (2000)'s Coasian Bargaining assumption, allow for suboptimal behavior of the resulting coalition. Kosfeld, Okada and Riedl (2009) examine the role of institution formation in providing a continuous public good while also allowing for the possibility that the institution (coalition) is rejected even if it is monetarily beneficial to its members. Burger and Kolstad (2010) report results from laboratory experiments that mimic the setup of a standard model of international environmental agreements, and study the relation between the benefit-cost ratio of providing public goods and participation behavior. None of these studies investigates the effect of group size on coalition formation.

Section 2 presents the voluntary participation game by Dixit and Olson (2000) with its equilibrium analysis and introduces a prior stage of communication. In Section 3, we design our experiments and present the major hypotheses for testing. Section 4 reports the experimental findings. Section 5 is devoted to an analysis with risk aversion. We conclude in Section 6. All proofs can be found in the Appendix.

2 Model and Equilibrium Analysis

2.1 Dixit and Olson (2000) and Further Analysis

N ex-ante identical players must decide whether or not to provide a lumpy public good. The benefit of the public good to each player is normalized to 1. The total cost of providing the public good is c . Assume $1 < c < N$, where the first inequality means that it is not individually rational for a single player to act alone and provide the public good, and the second implies that providing the public good is socially efficient. The game consists of two stages: in the first stage (participation stage), each player decides whether to participate in a coalition; in the second stage (provision stage), an efficient Coasian bargaining occurs in the resulting

coalition.

Dixit and Olson's (2000) efficient Coasian bargaining assumption simplifies the second-stage interactions. More precisely, denote the total number of members in the coalition by m . In the second stage, members engage in Coasian bargaining and the coalition will provide the public good if and only if $m \geq c$. If the public good is provided, each member of the coalition incurs a cost of $\frac{c}{m}$ and non-members take a free ride. Given this decision rule of the coalition, we say the coalition is successfully formed if and only if $m \geq c$.

Note that the Dixit and Olson (2000) game differs from the so-called step-level public good (SLPG) games (e.g., Palfrey and Rosenthal, 1984; see Rapoport, 1999 for a survey). A typical SLPG game is a one-stage game in which players decide whether to contribute a fixed amount and a public good is provided if the total sum of contributions is no less than an exogenous threshold. Contributions may be wasted if the threshold is not reached; meanwhile, over-provision of the public good is possible. In Dixit and Olson (2000), efficient Coasian bargaining excludes these possibilities.

Define $\lceil x \rceil$ as the smallest integer greater than or equal to x . Given the above decision rule for the coalition at the second stage, $\lceil c \rceil$ is the minimum required membership in the coalition to provide the public good. We say that a player is pivotal if and only if $\lceil c \rceil - 1$ other players join. When there are at least $\lceil c \rceil$ members in the coalition, a player's additional membership is superfluous from his/her own viewpoint: by joining the coalition, he/she incurs an additional cost of $\frac{c}{m}$ but does not affect whether the public good is provided. Like Dixit and Olson (2000), we rule out the trivial case of $N = \lceil c \rceil$. Dixit and Olson first look at pure strategy equilibria and have a result that can be summarized as follows.⁵

Remark 1. *There are two classes of pure-strategy Nash equilibria: (1) Less than $\lceil c \rceil - 1$ players join the coalition and no public good is provided and (2) $\lceil c \rceil$ players join the coalition and the resulting coalition provides the public good with non-members free-riding.*

The second class of equilibria with exactly $\lceil c \rceil$ participants in the coalition, however, arbitrarily requires identical players to choose different strategies in a precisely coordinated manner. It also conflicts with the non-cooperative nature of the first stage; the coordination problem is even more difficult in our context than in the corporate-takeover literature using mixed strategies (e.g., Bagnoli and Lipman, 1988; Holmstrom and Nalebuff, 1992) because "the potential participants are not even identified until they show up for the meeting" (Dixit and Olson 2000, p.318). Therefore, Dixit and Olson turn their attention to a symmetric mixed strategy equilibrium. In the rest of this section, we also focus on mixed strategies in the first

⁵We simplify Dixit and Olson's (2000) notations by normalizing the per capita benefit of the public good V in their model to 1 and using $\lceil c \rceil$ to denote M in their model.

stage. In the symmetric mixed strategy equilibrium, each player joins the coalition with probability p . The equilibrium condition for p is

$$\frac{(N-1)!}{([\![c]\!] - 1)!(N - [\![c]\!])!} p^{[\![c]\!] - 1} (1-p)^{N - [\![c]\!] } \left(1 - \frac{c}{[\![c]\!]}\right) = \sum_{i=[\![c]\!]}^{N-1} \frac{(N-1)!}{i!(N-1-i)!} p^i (1-p)^{N-1-i} \frac{c}{1+i} \quad (1)$$

where the left-hand side is the expected benefit of being pivotal and the right-hand side is the expected cost of being superfluous. The Appendix shows that Equation (1) could be rewritten as

$$\frac{\frac{N!}{[\![c]\!]!(N - [\![c]\!])!} p^{[\![c]\!] } (1-p)^{N - [\![c]\!] }}{\sum_{j=[\![c]\!]}^N \frac{N!}{j!(N-j)!} p^j (1-p)^{N-j}} = \frac{c}{[\![c]\!]}. \quad (2)$$

This equation is equivalent to the equilibrium condition, Equation (6), in Dixit and Olson (2000). The inverse of the left-hand side of (2) is a hypergeometric function, *hypergeom* $\left([1, -N + [\![c]\!], 1 + [\![c]\!], \frac{p}{-1+p}\right)$.⁶ A hypergeometric function, *hypergeom* $([b, a], v, z)$ satisfies

$$\text{hypergeom}([b, a], v, z) \equiv \frac{\Gamma(v)}{\Gamma(b)\Gamma(v-b)} \int_0^1 \frac{t^{b-1} (1-t)^{v-b-1}}{(1-tz)^a} dt. \quad (3)$$

Using this and the fact that the gamma function $\Gamma(x+1) = x!$ when x is an integer, the left-hand side of (2) could be rewritten as

$$\frac{\frac{N!}{[\![c]\!]!(N - [\![c]\!])!} p^{[\![c]\!] } (1-p)^{N - [\![c]\!] }}{\sum_{j=[\![c]\!]}^N \frac{N!}{j!(N-j)!} p^j (1-p)^{N-j}} = \frac{1}{[\![c]\!] \int_0^1 (1-t)^{[\![c]\!] - 1} \left(1 + t \frac{p}{1-p}\right)^{N - [\![c]\!] } dt}. \quad (4)$$

Equation (2) therefore becomes⁷

$$c \int_0^1 (1-t)^{[\![c]\!] - 1} \left(1 + t \frac{p}{1-p}\right)^{N - [\![c]\!] } dt = 1. \quad (5)$$

We hereby express the equilibrium condition in a continuous form. This equation will be useful to obtain some of the analytical results beyond Dixit and Olson (2000).⁸

⁶This result and Equation (3) are confirmed by the mathematical software *Maple*. For generalized hypergeometric functions, see <http://mathworld.wolfram.com/HypergeometricFunction.html>.

⁷Alternatively, we can use the mathematical identity in Wang (1994),

$$\sum_{i=k}^n \frac{n!}{i!(n-i)!} p^i (1-p)^{n-i} \equiv \frac{n!}{(k-1)!(n-k)!} \int_0^p t^{k-1} (1-t)^{n-k} dt,$$

to derive Equation (5). The Online Appendix A provides the details.

⁸Hong and Karp (2012, 2014) characterize the mixed strategy equilibrium of a linear International Environmental Agreement (IEA) game. Hong and Karp (2012) also use the properties of hypergeometric functions. The game of Dixit and Olson (2000) is different from the IEA game in that there is only one lumpy public good to be provided in the former, while each country can provide one unit of public good in the latter.

When c is an integer, i.e., $\lceil c \rceil = c$, a pivotal player obtains a benefit of one unit from the public good by joining the coalition but incurs a one unit cost as well, so the benefit of being pivotal is zero. Moreover, when there are at least $\lceil c \rceil$ members in the coalition, an additional participant incurs costs without changing the behavior of others and, more importantly, without affecting whether or not the public good is provided. Therefore, no individual player is incentivized to participate in the coalition when c is an integer.⁹ We thus have the following result, which complements the analysis in Dixit and Olson (2000).

Remark 2. *When c is an integer, $p = 0$.*

Although the result on c being an integer is of theoretical interest, it is of little practical relevance since the costs being exactly an integer is a probability 0 event. We thus focus on the case when c is not an integer. Part (i) of the following proposition formally establishes the existence and uniqueness of a symmetric mixed strategy equilibrium when c is not an integer, which was assumed in Dixit and Olson (2000).

Proposition 1. *(i) When c is a non-integer, there is a unique (non-degenerated) symmetric mixed strategy equilibrium with $p(c, N) \in (0, 1)$. (ii) For any interval in which $\lceil c \rceil$ is a constant, p decreases in c .*

The intuition of part (ii) of this proposition is the following. When $\lceil c \rceil$ is fixed, an increase in c reduces the benefit of being pivotal, which makes participation less attractive; as a result, the equilibrium participation probability is lower. Proposition 1, Remark 2 and the continuity of the function in the left-hand side of Equation (5) given constant $\lceil c \rceil$ imply that when c approaches an integer from below, where $\lceil c \rceil$ is held constant, the equilibrium p will decrease and converge to 0 in a continuous manner, but the equilibrium p will jump upwards discontinuously when c increases just above the integer, where $\lceil c \rceil$ increases by one. This discontinuity is driven by the discontinuity of the benefit of being pivotal, $\left(1 - \frac{c}{\lceil c \rceil}\right)$, when $\lceil c \rceil$ increases by one.

Using simulations, Dixit and Olson (2000) find that the equilibrium participation probability and thus expected membership are typically low, resulting in a very low chance of successful provision of the public good. Define expected membership $\bar{m} = pN$. We have the following new analytical result regarding the expected membership, which is not precisely discussed in Dixit and Olson (2000):

Proposition 2. *When $c \geq 2$, the expected membership of the mixed strategy equilibrium, \bar{m} , is less than $\lceil c \rceil$.*

⁹Mathematically, if c is an integer ($\lceil c \rceil = c$), the left-hand side of (1) becomes 0. Thus, we must have $p = 0$ such that the right-hand side of (1) is 0.

Proposition 2 provides a pessimistic insight for the case of $c \geq 2$: under the mixed strategy equilibrium, the expected coalition membership is lower than the critical size at or above which the public good is provided, implying that it is likely that the public good is not provided. When $1 < c < 2$, however, it is possible that \bar{m} is greater than 2, the corresponding critical size at or above which the public good is provided. For example, for $N = 3$, when c approaches 1 from the right, p goes to 0.75 (according to Equation (5)), and thus \bar{m} goes to 2.25, which is greater than 2.

Dixit and Olson’s (2000) *numerical* analysis provides a comparative statics result on individual participation and coalition formation with respect to the group size. They argue that “as N increases, the probability that any one individual chooses IN decreases” (p.323). This observation provides an important background for their major conclusion that voluntary participation undermines the Coase theorem, especially for larger groups. We prove this analytically in the following lemma.

Lemma 1. *p is decreasing in N .*

As N increases, for a given p , a player’s membership becomes more likely to be superfluous from her own viewpoint. To maintain indifference between joining and not joining in the coalition, p should therefore decrease. As a further step, we obtain the following new result on the expected membership.

Proposition 3. *When $c \geq 2$, \bar{m} is decreasing in N .*

Dixit and Olson (2000) argue that the probability that the public good is provided decreases in N (p.323). Our additional simulations lend support to this argument, although we can only prove this result for the case of $0 < c < 1$. Let $G_1(N)$ denote the probability that the public good is provided for $0 < c < 1$.

Remark 3. *$\frac{dG_1(N)}{dN} < 0$, i.e. the probability that the public good is provided by the coalition is decreasing in the number of players, when $0 < c < 1$.*

2.2 Pre-play Communication

To capture the essence of coordination improvement through cheap talk among players, consider the following stylized model. Before players make their participation decisions, players independently and simultaneously send group members a non-binding and non-verifiable message $s \in \{\text{“yes”}, \text{“no”}\}$ that has no direct payoff implication.¹⁰ After observing the messages

¹⁰The binary message space is not necessary in our model. Any message space with cardinality larger than that of the binary message space will lead to essentially the same result.

from others, each player decides simultaneously whether to participate in the coalition. One can interpret the message “yes” (“no”) as an intention to (not to) participate in the coalition. Except for the introduction of the pre-play communication stage, all other components of the game remain the same.

In the extended game with pre-play communication, there exists an equilibrium in which the coalition is formed with probability strictly greater than that of the symmetric mixed-strategy equilibrium of the original game without communication. Consider the following strategy profile. (1) Each player randomizes between “yes” and “no” with probabilities q and $1 - q$, respectively. (2) If the total number of “yes” messages, denoted by N_y , is $\lceil c \rceil$, then those who sent “yes” participate in the coalition with probability 1 and those who sent “no” participate in the coalition with probability 0. (3) If $N_y = N - \lceil c \rceil$, then those who sent “no” participate in the coalition with probability 1 and those who sent “yes” participate in the coalition with probability 0. (4) If N_y is neither $\lceil c \rceil$ nor $N - \lceil c \rceil$, then players participate with the mixed strategy that they would use in the symmetric mixed-strategy equilibrium of the original game without communication.

Let EX denote the expected payoff from the second-stage mixed-strategy participation equilibrium. Let $C_n^k = \binom{n}{k}$ denote the binomial coefficient. Observe that when everyone sends “yes” with probability q , the expected payoff from sending “yes” is

$$EU(\text{“yes”}) = C_{N-1}^{\lceil c \rceil - 1} q^{\lceil c \rceil - 1} (1 - q)^{N - \lceil c \rceil} \left(1 - \frac{c}{\lceil c \rceil} \right) + C_{N-1}^{N - \lceil c \rceil - 1} q^{N - \lceil c \rceil - 1} (1 - q)^{\lceil c \rceil} \\ + \left(1 - C_{N-1}^{\lceil c \rceil - 1} q^{\lceil c \rceil - 1} (1 - q)^{N - \lceil c \rceil} - C_{N-1}^{N - \lceil c \rceil - 1} q^{N - \lceil c \rceil - 1} (1 - q)^{\lceil c \rceil} \right) EX \quad (6)$$

whereas the expected payoff from sending “no” is

$$EU(\text{“no”}) = C_{N-1}^{N - \lceil c \rceil} q^{N - \lceil c \rceil} (1 - q)^{\lceil c \rceil - 1} \left(1 - \frac{c}{\lceil c \rceil} \right) + C_{N-1}^{\lceil c \rceil} q^{\lceil c \rceil} (1 - q)^{N - 1 - \lceil c \rceil} \\ + \left(1 - C_{N-1}^{N - \lceil c \rceil} q^{N - \lceil c \rceil} (1 - q)^{\lceil c \rceil - 1} - C_{N-1}^{\lceil c \rceil} q^{\lceil c \rceil} (1 - q)^{N - 1 - \lceil c \rceil} \right) EX. \quad (7)$$

When $q = \frac{1}{2}$, it is immediate that the two expressions above become the same; thus, the above strategy profile constitutes a Nash equilibrium.

In this equilibrium, the cheap talk messages play the role of a pure coordination device.¹¹ In particular, when $N_y = \lceil c \rceil$ (or $N - \lceil c \rceil$), players coordinate on the pure strategy Nash equilibrium in which the coalition is certainly formed. Therefore, the *ex-ante* probability of successful coalition formation (or, in other words, provision of the public good) in this equilibrium

¹¹See, e.g., Farrell (1987) for a reference on the role of cheap talk in coordination. Related approaches include Myerson (1986) (communication equilibrium) and Aumann (1974) (correlated equilibrium). Farrell and Rabin (1996) provide a non-technical survey on cheap talk.

is strictly higher than that in the mixed-strategy equilibrium of the original game without communication. This result is summarized in the following proposition.

Proposition 4. *In the extended game with pre-play communication, there exists an equilibrium in which the ex-ante probability of successful coalition formation is strictly higher than that in the unique symmetric mixed-strategy equilibrium of the original game without communication.*

It is worth noting that there are other equilibria for the game. In the above equilibrium construction, sending the message “yes” does not differ from sending the message “no” as an expression of the intention of participation. However, language has its literal meanings. If a message “yes” (“no”) is interpreted as intention to (not to) participate, players are more likely to coordinate on the pure strategy participation equilibrium when $N_y = \lceil c \rceil$ than when $N_y = N - \lceil c \rceil$. Under an alternative strategy profile in which players coordinate on the outcome of coalition formation only when $N_y = \lceil c \rceil$, the equilibrium probability of sending “yes” would be different from $\frac{1}{2}$. In this equilibrium, the probability of successful coalition formation is also higher than that in the unique symmetric mixed-strategy equilibrium of the original game without communication. Meanwhile, there exists a babbling equilibrium in which no information is transmitted via cheap talk and thus players completely ignore the messages.

3 Experimental Design

3.1 Design and Hypotheses

Fixing $c = 2.5$, we control for the group size and the presence/absence of the pre-play communication, which results in the 2×2 design in Table 1.

Table 1: Experimental Treatments

Treatment/Game	Group Size	p	\bar{m}	G	
Baseline Treatments	$B4$	4	0.444	1.778	0.234
	$B8$	8	0.125	0.998	0.067
Communication Treatments	$C4$	4	0.597	2.389	0.617
	$C8$	8	0.234	1.874	0.475

Note: This table presents the 2×2 experimental design, and the theoretical predictions based on the equilibrium constructions in Section 2.

In this table, p , \bar{m} and G denote the *ex-ante* probability of participation, expected membership and the probability of successful provision of the public good, respectively. The table

also presents the theoretical predictions based on the mixed-strategy equilibrium constructions in Section 2. The numbers shown for the communication treatments are those of the equilibrium in which the cheap talk messages “yes” and “no” play as coordination devices as discussed before.

Proposition 2 and Table 1 predict that, with $c = 2.5$, the expected membership is lower than 3 and therefore the public good is not likely to be provided. We thus have the following hypothesis:

Hypothesis 1. *The membership in the treatments without communication is lower than 2. The likelihood of successful coalition formation is less than 25% in both treatments without communication.*

Lemma 1, Proposition 3 and the table above lead to the following hypothesis.

Hypothesis 2. *Controlling for communication, both the participation level and the likelihood of successful coalition formation with $N = 8$ are strictly less than those with $N = 4$.*

Our third hypothesis revolves around the role of communication.

Hypothesis 3. *Controlling for the group size, communication increases both the participation level and the likelihood of successful coalition formation.*

3.2 Experimental Procedures

All of the treatments share the same basic experimental procedures. Three sessions are conducted for each treatment using the between-subject design. All sessions are conducted in Chinese using z-Tree (Fishchbacher, 2007) in the experimental laboratory at the Shanghai University of Finance and Economics in April 2012. A total of 264 subjects, primarily freshman and sophomore undergraduate students with some law-school master’s students, none of whom have any prior experience with our experiment, were recruited from the university.

For each session, games with $N = 4$ are implemented for 5 groups, whereas games with $N = 8$ are implemented for 3 groups.¹² Upon arrival, subjects are instructed to sit at separate computer terminals. Each receives a copy of the experimental instructions, which are also read aloud by an experimenter.

¹²As a result, the number of subjects in a session with $N = 4$ is 20 and the number of subjects in a session with $N = 8$ is 24.

For illustration purposes, we will detail the game with communication where $N = 8$. More details can be found in the instructions presented in the Online Appendix.¹³ In each session, subjects form a group of 8 and participate in 30 rounds. The “Stranger” matching protocol is used. In each round, each player is asked to make two decisions according to the following two stages:

1. Communication Stage¹⁴: At the beginning of each round, the computer asks each player to send a message to the group members about his/her intention to participate in **Coalition C** by clicking “Yes” or “No”. The “Yes” or “No” choice does not directly affect players’ payoffs. The total number of people in the group who chose “Yes” is announced to everyone in the group.
2. Participation Stage: After observing the total number of “Yes” messages in the group, each player is asked to decide whether to be a member of **Coalition C**. The player is free to choose to (or not to) participate in **Coalition C**, regardless of his/her message choice in the communication stage. After all individuals in the group make their decisions, the total number of members, denoted by m , of **Coalition C** is publicly announced.¹⁵

The earnings in each round are determined as follows. If the total number of members of Coalition C, m , is strictly greater than two, then the coalition is formed and everyone in the group will receive 100, while each member of Coalition C pays $\frac{250}{m}$. Otherwise, the coalition is not formed and no one in the group would gain or lose anything.

We add one more stage to elicit beliefs of subjects before they make the participation decisions. The computer asks “How many people (excluding yourself) in your group do you believe will participate in Coalition C?” Players are free to choose any integer number between 0 and 7 (or between 0 and 3 for the $N = 4$ treatments). This choice does not affect their earnings and is never revealed to other participants.¹⁶

A payoff of 5 translates into a real payment of 1 RMB yuan. A subject is paid the sum of rewards from three randomly selected rounds plus 10 RMB yuan for participating. One

¹³An English translation of the experimental instructions for Treatment C8 can be found in the Online Appendix B. The original instructions for Treatment C8 written in Chinese are also provided in the Online Appendix C.

¹⁴This stage is absent in the baseline treatments.

¹⁵In the real instructions, we used N to denote the coalition membership.

¹⁶We do not incentivize subjects to report their beliefs because 1) any simple elicitation mechanism such as a quadratic scoring rule cannot provide risk-averse subjects with a truth-telling incentive and thus 2) a rather complicated incentive mechanism is necessary to make it incentive-compatible for subjects to report their belief truthfully. Another potential problem of paying for beliefs is that it may create hedging incentives when subjects earn money both by the actions they take in the experiment and from their elicited beliefs. For an experimental investigation of the hedging problem, see Blanco, Engelmann, Koch and Normann (2010).

session lasted about 1.5 hours. Subjects earned an average of 26.7 RMB yuan (\approx US\$ 4.3), ranging from 10 RMB yuan (\approx US\$ 1.6) to 70 RMB yuan (\approx US\$ 11.1).¹⁷

4 Experimental Findings

4.1 Main Findings

Figure 1 and Figure 2 respectively present the frequency of successful coalition formation and the average membership aggregated across all rounds over all sessions.¹⁸ In each session, as predicted by Dixit and Olson (2000) and consistent with our Proposition 2, the observed frequency of the successful public goods provision falls short of 100% and the average membership never reaches 3.

A more careful look at the data, however, tells us that without communication, the observed frequencies of the coalition formation and the average memberships are higher than the theoretical predictions. In treatment *B4*, the observed frequency of coalition formation is 0.442 and the observed average membership is 2.34, higher than the theoretical predictions of 0.234 and 1.778, respectively; in treatment *B8*, the observed frequency of the coalition formation and the observed average membership are 0.519 and 2.56, higher than the theoretical predictions of 0.067 and 0.998, respectively. Moreover, using the session-level aggregate average data, the sign tests (Snedecor and Cochran, 1989) reveals that we can reject Hypothesis 1 ($p < 0.05$), which states that average membership is less than 2 and the likelihood of coalition formation is less than 25%. The higher frequencies of coalition formation are direct consequences of higher frequencies of individual participation, as presented in Figures 3(a) and (b). Subjects tend to participate in the coalition almost 60% of the time in treatment *B4* and over 30% of the time in treatment *B8*. This pattern appears clearly from the early rounds and seems to be stable over rounds.

Result 1. *Although it is likely that the public good is not provided, the observed frequencies of participation, coalition formation and average memberships in the baseline treatments are significantly higher than the theoretical predictions.*

There are several noticeable differences between treatments with $N = 4$ and treatments with $N = 8$. On the one hand, consistent with the theoretical prediction (Lemma 1), the non-parametric Mann-Whitney U tests at the session level ($p < 0.01$) reveal that the frequency

¹⁷Although the average payment is smaller than the standard amount paid to subjects in the US, we believe that the amount is sufficiently big to motivate subjects. A regular meal in the university cafeteria costs approximately US\$ 1 only.

¹⁸All other bar charts in this paper also use frequencies aggregated across all rounds over all sessions.

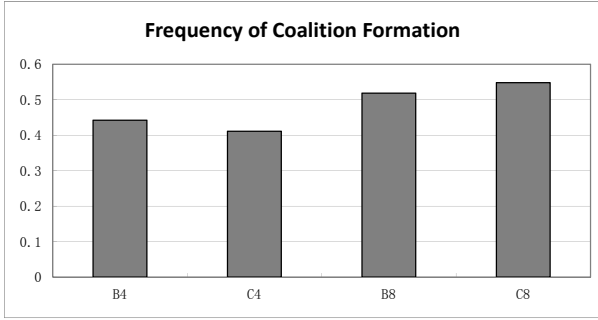


Figure 1: Coalition Formation

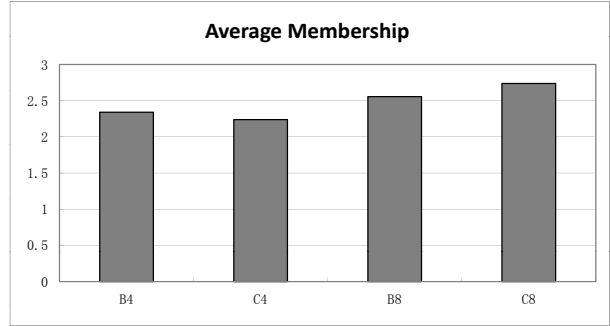
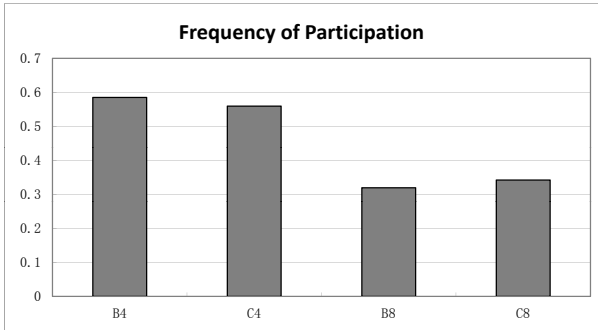
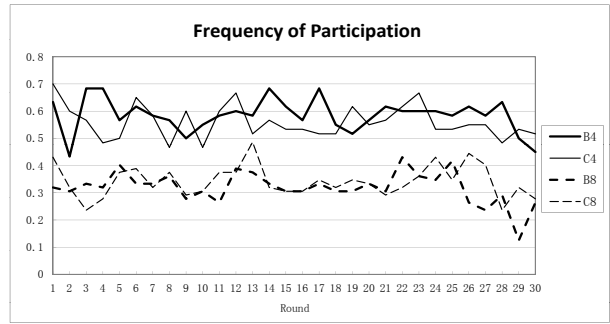


Figure 2: Average Membership



(a) Average Frequency



(b) Time Trend

Figure 3: Frequency of Participation

of participation under $N = 4$ is significantly higher than that under $N = 8$; on the other hand, both the frequency of coalition formation and the average membership under $N = 4$ are significantly lower than those under $N = 8$ ($p < 0.01$ and $p < 0.05$, Mann-Whitney U tests). This result is another sharp contradiction to the theoretical predictions in Proposition 3 and in Dixit and Olson (2000). We thus reject Hypothesis 2, which states that the participation level and the likelihood of successful coalition formation with $N = 8$ are strictly lower than those with $N = 4$.

Result 2. *The observed frequencies of participation are significantly higher when $N = 4$ than when $N = 8$, while the observed frequencies of coalition formation and the average membership are significantly lower when $N = 4$ than when $N = 8$.*

We conduct a regression analysis to look into more details of the participation behavior, using the individual-level data. Table 2 reports *odds ratios* from logit regressions, with a binary variable indicating participation as a dependent variable. We create a dummy variable, *Pivot*, to indicate whether or not an individual believes that 2 others in the group will participate in the coalition and another dummy variable, *Superfluous*, to indicate whether or not the individual believes that at least 3 others in the group will participate in the coalition. In all columns of Table 2, we control for time trend and treatment effects by including

Table 2: Logit Regression Results on Participation

	(1)	(2)	(3)	(4)	(5)	(6)
<i>N8</i>	0.4045*** (0.0477)	0.2875*** (0.0937)	0.4039*** (0.0297)	0.4061*** (0.0495)	0.2751*** (0.0925)	0.4047*** (0.0361)
<i>Communicate</i>	0.9670 (0.1139)	1.182 (0.3481)	0.9641 (0.0707)	0.9511 (0.1137)	1.218 (0.3647)	0.9465 (0.0835)
<i>Pivot</i>	2.246*** (0.1991)	2.254*** (0.2009)	2.251*** (0.1121)			
<i>Belief</i>				1.434*** (0.1365)	1.409*** (0.1267)	1.417*** (0.0719)
<i>Superfluous</i>				0.3453** (0.1300)	0.3082*** (0.1139)	0.3191*** (0.0609)
<i>Superfluous</i> × <i>Belief</i>				0.8851 (0.1211)	0.9179 (0.1214)	0.9077 (0.0651)
Session Effects Controlled for	No	Yes	No	No	Yes	No
Session-level RE Model	No	No	Yes	No	No	Yes
Pseudo R^2	0.0718	0.0744	N.A.	0.0832	0.0869	N.A.
No. of Observations	7920	7920	7920	7920	7920	7920

Note: This table reports odds ratios from logit regressions on participation, controlling for period effects. Standard errors are reported in parentheses. In Columns (1), (2), (4) and (5), standard errors are clustered at the individual level. Columns (2) and (5) control for session effects, while Columns (3) and (6) estimate the random effect model at the session level. *** for $p < 0.001$, ** for $p < 0.01$, and * for $p < 0.05$.

dummy variables of periods, as well as *N8* and *Communicate*, which indicate whether group size is 8 and whether communication is allowed respectively. In the first three columns, as right-hand-side variables, we include *Pivot*, while in the last three columns, we include *Belief* (on the number of other participants), *Superfluous* and their interaction term. For Columns (1)-(2) and (4)-(5), standard errors are clustered at the individual level. Columns (2) and (5) further control for dummy variables of sessions. Columns (3) and (6) report results from panel logit regressions with random effects at the session level.

Table 2 reports that, confirming the finding in Result 2, players in treatments with $N = 8$ are significantly less likely to participate than in treatments with $N = 4$ (odds ratios ≈ 0.3 - 0.4), after controlling for other variables. Moreover, across different specifications, subjects who believe two others participate ($Pivot = 1$) are significantly more likely to participate than otherwise (odds ratios ≈ 2.3), and players who believe at least three others participate ($Superfluous = 1$) are significantly less likely to participate than otherwise (odds ratios ≈ 0.3).¹⁹ These results evidently demonstrate that subjects understand the incentives of being pivotal and of free-riding. Moreover, Columns (4)-(6) of Table 2 show that, when *Superfluous* is controlled for, participation is significantly more likely as *Belief* increases (odds ratios > 1). However, the effect of the interaction term between *Belief* and *Superfluous* is not statistically

¹⁹However, we do observe that some subjects with $Pivot = 1$ do not participate and some subjects with $Superfluous = 1$ do participate. This may be related to the way we elicit beliefs. In our design, it is not obvious that players reporting that 3 others participate will not participate because if people state the mean of their belief, they might, for example, believe that with probability 1/2 2 others participate and with probability 1/2 4 others participate. It pays to participate in the first case, and so it might do on expectation.

significant under any of the specifications, and the estimated odds ratios are close to 1. The above results imply that there is a *Belief* effect even when *Superfluous*=1.²⁰

Figure 4 presents the participation frequencies given different beliefs about others' participation and Figure 5 reports the distribution of beliefs. Consistent with the regression results, Figure 4(a) shows that the average participation likelihoods in treatment *B8* are weakly increasing in *Belief* conditional on $Belief > 2$.²¹ We call this observation *conditional participation*: A player is more likely to participate if he believes that more of his peers will participate.²² This behavioral pattern may be driven by social preferences, such as fairness, in public goods provision.²³ If one believes that more people are to participate in the coalition, the expected cost of participating in the coalition and being superfluous is lower. This implies that the net benefit from taking a free ride becomes smaller so that it is possible for the fairness concerns to dominate the free-riding incentives. As a result, individuals with fairness concerns are more likely to participate when they have more optimistic beliefs on others' participation.

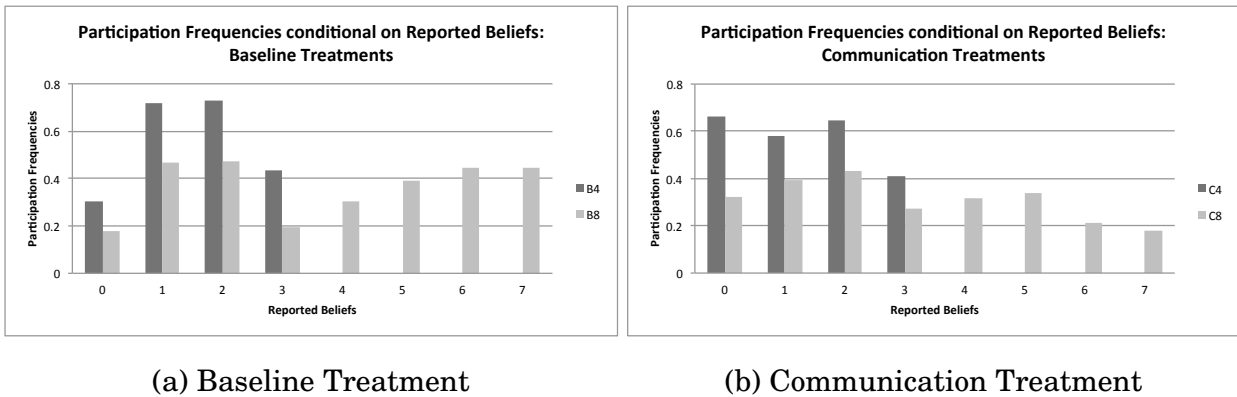


Figure 4: Frequencies of Participation Conditional on Belief about the Number of Other Participants

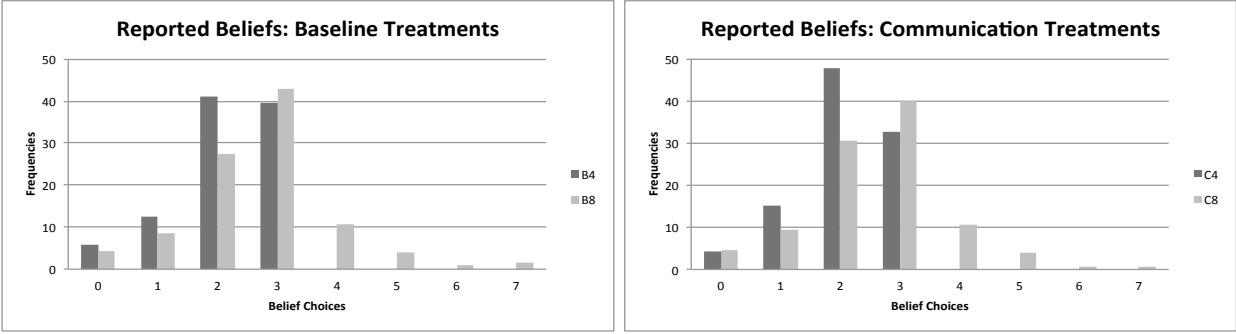
Now we explain the comparative statics on coalition formation as our Result 2 reports. Increasing the group size N creates two opposing effects on coalition formation. The direct effect is that, fixing the individual participation probability, the larger group size increases

²⁰In the Online Appendix D, we also report results from the regressions separately conducted for $N = 4$ (Table 7) and $N = 8$ (Table 8) treatments. The results show that there is no qualitative difference between the separate regressions and the aggregated regression. In Table 7, the interaction term $Superfluous \times Belief$ is dropped because with $N = 4$, when $Superfluous = 1$, $Belief$ can only take one value, which is 3.

²¹However, this trend does not hold for treatment *C8*. For treatment *C8*, the participation frequencies become lower when $Belief > 5$, but subjects with $Belief > 5$ account for less than 1.2% of observations for the treatment *C8*.

²²Fischbacher et al. (2001) and Frey and Meier (2004) find evidence of *conditional cooperation* in public goods provision. See Section 2 of Chaudhuri (2011) for a review of the literature, where conditional cooperators are defined as those “whose contribution to the public good is positively correlated with their beliefs about the contributions to be made by their group members” (p. 49).

²³See, e.g., Fehr and Gächter (2000) and Kosfeld et al. (2009).



(a) Baseline Treatments

(b) Communication Treatments

Figure 5: Distribution of Reported Beliefs on the Number of Other Participants

the chance of coalition formation. The indirect effect is that it leads to a reduction in the individual participation probability because each individual is more likely to be superfluous. The theoretical analysis suggests that the indirect effect should dominate the direct effect and, consequently, the coalition is less likely to be formed with the larger group size. However, the empirically observed indirect effect, the decrease in participation probability when $N = 8$, is too small to dominate the direct effect.

The conditional participation may account for this result. When $N = 8$, a substantial proportion (16.4%) of players believe that at least 4 other players in their group would participate – trivially, these players are absent under $N = 4$ – and these players on average participate in the coalition 33% of the time.²⁴ Note that a theory that does not take into account social preferences predicts that players with such beliefs would not join the coalition with probability 1. This disparity may be able to explain why the decrease in participation probability is not as large as the theoretical predictions when N increases from 4 to 8.

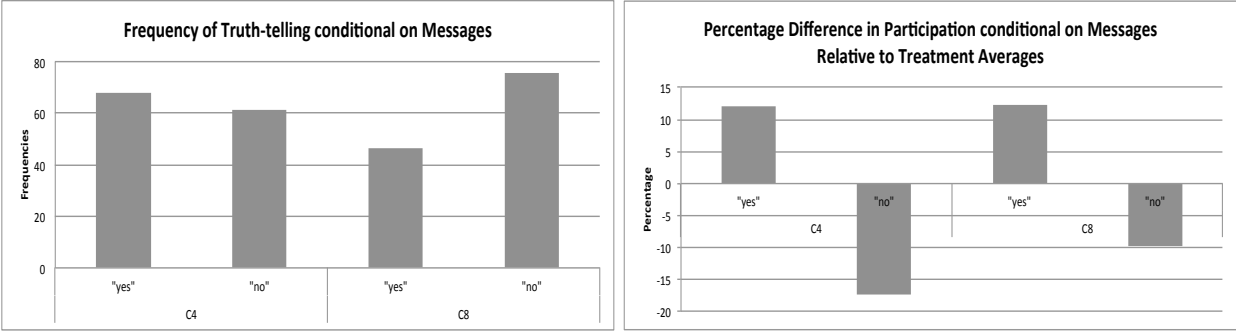
Our experimental findings thus suggest a more optimistic view about voluntary participation in the public goods provision, particularly when more people are involved.

4.2 Other Findings: Communication

Table 2 reports that the communication has no statistically significant effect on individuals' participation likelihood. This subsection reports more observations regarding communication.

Figure 6(a) reports the frequencies of truth-telling defined as the outcome in which the

²⁴Kosfeld et al. (2009) allows for the rejection of a resulting coalition by participants even if the coalition is monetarily beneficial to participants. In their experiment, many players believe that all other players in the group will participate, and these players are likely to participate, which leads to grand coalitions at times.



(a) Truth-telling

(b) Difference in Participation Frequencies

Figure 6: Truth-telling Frequencies

literal meaning of the chosen message coincides with the participation decision made. Figure 6(b) shows to what extent an individual sending message “yes” (“no”) is more (less) likely to participate in the coalition compared to the average level of participation in the treatment. Specifically, the positive bars report the participation level conditional on message “yes” and the negative bars report the participation level conditional on message “no” when the unconditional average participation level in the treatment is normalized to 0%. The figure indicates that individuals who chose the “yes” message participated in the coalition 29.3% in treatment C4 and 22% in treatment C8 *more often* than those who chose the “no” message. These figures suggest that communication is informative.

Table 3: Frequencies of Participation and Coalition Formation

Treatment	Session	Participation		Coalition Formation	
		$N_y = 3$	$N_y \neq 3$	$N_y = 3$	$N_y \neq 3$
C4	1	.40	.35	.75	.53
	2	.63	.58	.53	.41
	3	.62	.53	.48	.38
C8	1	.31	.34	.38	.59
	2	.58	.50	.43	.34
	3	.31	.34	.57	.52
Total		.48	.42	.52	.47

Note: This table reports the frequencies of participation and coalition formation at the session level, when $N_y = 3$ and when $N_y \neq 3$, respectively.

There is some evidence that communication facilitates coordination in a way that is partially consistent with the constructed equilibrium with communication. We present the evidence at the session and group levels. Table 3 reports the frequencies of participation and coalition formation at the session level, when the announced number of “yes” (N_y) is three and when $N_y \neq 3$, respectively. In five out of the six sessions with pre-play communication, the frequency of coalition formation when $N_y = 3$ is higher than that with $N_y \neq 3$. In particular,

in the session 3 of C8, the participation frequency with $N_y = 3$ is lowest among all sessions whereas the frequency of coalition formation with $N_y = 3$ is second highest, which suggests that better coordination among players has been achieved when $N_y = 3$. A one-sided sign test (Snedecor and Cochran, 1989) reveals that the median of coalition formation frequencies with $N_y = 3$ is insignificantly greater than that with $N_y \neq 3$ ($p = 0.109$).

We further regress a binary variable indicating successful coalition formation on a dummy variable indicating $N_y = 3$ using the group-level data. Table 4 presents *odds ratios* from the logit regressions with time trend controlled for by period dummy variables. Columns (1)-(2) report results from regressions using all data from treatments with communication and controlling for treatment variable $N8$, while Columns (3)-(4) and (5)-(6) report results from regressions using data from C4 and C8 treatments respectively. Columns (1), (3) and (5) also control for session effects and report robust standard errors, while the other three columns estimate the panel logit model with random effects at the session level.²⁵ Confirming the finding in Result 2, the first two columns of Table 4 report that a group in treatment C8 is significantly more likely to form the coalition successfully than in treatment C4 (odds ratios ≈ 2), after controlling for other variables. Overall, $N_y = 3$ increases the likelihood of coalition formation at a marginally significant level (odds ratios ≈ 1.4).²⁶ Meanwhile, this positive effect of $N_y = 3$ is statistically significant for treatment C4 but not for treatment C8, suggesting that communication perhaps facilitates coordination better for smaller groups. The results from Table 4 suggest that communication is sometimes self-enforcing, along the lines of Farrell and Rabin (1996). However, we do not find evidence that players coordinate in participation when $N_y = N - 3$.

Overall, there is weak evidence that communication increases membership and welfare. The average membership is higher with communication (2.51 vs. 2.46), as is the average round payoff (27.4 vs. 26.8). Mann-Whitney U tests show that there is no significant difference between the baseline treatments and the communication treatments in participation level and the frequency of coalition formation.

Result 3. *Although the observed average membership and average round payoff are higher with communication, there is no significant evidence suggesting that communication improves participation and the frequency of coalition formation.*

²⁵In Columns (1), (3) and (5), clustering standard errors at the session level does not change the results qualitatively, except that in Columns (1) and (3), the odds ratios of $N_y = 3$ will become more statistically significant. The results are reported in Table 9 of the Online Appendix D.

²⁶The p value for $N_y = 3$ in Column (1) is 0.05.

Table 4: Coalition Formation: Communication Treatments

	Total		$N = 4$		$N = 8$	
	(1)	(2)	(3)	(4)	(5)	(6)
N8	2.100** (0.5810)	1.849*** (0.2977)				
$N_y = 3$	1.398 (0.2386)	1.402* (0.2409)	1.546* (0.3306)	1.563* (0.3326)	1.205 (0.3754)	1.191 (0.3863)
Session Effects Controlled for	Yes	No	Yes	No	Yes	No
Session-level RE Model	No	Yes	No	Yes	No	Yes
Pseudo R^2	0.0570	N.A.	0.0559	N.A.	0.0988	N.A.
No. of observations	720	720	450	450	270	270

Note: Using group level data, this table reports odds ratios from logit regressions on coalition formation, controlling for period dummy variables. Standard errors are in parentheses. Columns (1), (3) and (5) control for session dummy variables, with robust standard errors reported, while Columns (2), (4), and (6) estimate the random effect model at the session level. *** for $p < 0.001$, ** for $p < 0.01$, and * for $p < 0.05$.

5 Discussion: Over-participation and Risk-aversion

This section discusses the high participation frequencies reported in Result 1 and provides a rationale based on subjects' risk attitude.^{27,28} Note that the use of mixed strategies creates endogenous strategic uncertainty because the membership of the coalition may or may not reach the threshold above which the public good is provided. Thus, it is immediate that the assumption on players' risk attitude affects the equilibrium outcome.

We depart from Dixit and Olson (2000) by assuming that players are risk averse.²⁹ Let $U(\cdot)$ be a concave function, strictly concave in some interval within $\left[1 - \frac{c}{|c|}, 1\right]$. The following proposition shows that risk-aversion results in an outcome that is not as pessimistic as Dixit and Olson (2000) predict.

Proposition 5. *For non-integer values of c , introducing the risk aversion increases the equilibrium participation probability.*

²⁷Being pivotal is crucial in subjects' participation decisions and thus it is possible that "over-participation" arises from subjects' over-estimation of the probability of being pivotal. Figure 5 shows that 44.9% ($N = 4$) and 29% ($N = 8$) of subjects report that they believe they are pivotal. The reported values are significantly higher than the theoretical probabilities of being pivotal in the mixed strategy equilibrium of the game, which are approximately 33% when $N = 4$ and 17% when $N = 8$. However, it appears that the subjects have somewhat accurate beliefs about the chance of being pivotal based on the empirically observed parameters. Given the average participation frequencies of 0.572 ($N = 4$) and 0.331 ($N = 8$), the likelihoods of being pivotal are 42% ($N = 4$) and 30.8% ($N = 8$), respectively.

²⁸Cason et al. (2004) find high level of participation due to the spiteful behavior in the second stage in which participants choose suboptimal contributions. In our experiment, we rule out this possibility by imposing Coasian bargaining in the second stage.

²⁹Evidence shows that players are risk averse in reality, as well as in the laboratory. See, e.g., Holt and Laury (2002), Goeree et al. (2002, 2003), and Harrison and Rutström (2008). Goeree et al. (2002) argue that players are risk averse over income defined relative to a specific gamble or small time unit.

The intuition behind Proposition 5 is straightforward. Note that for both members and non-members the income weakly increases with the number of members in the coalition. Under risk-aversion, the marginal utility of income decreases as income increases. A player loses $\frac{c}{i+1}$ units of income by joining the coalition if $i \geq [c]$ other players have joined, in which case the player's income is $1 - \frac{c}{i+1}$. The larger i is, the greater the income. A player gains $1 - \frac{c}{[c]}$ by joining the coalition only if exactly $[c] - 1$ other players join, in which case the player's income is low ($1 - \frac{c}{[c]}$) and thus its marginal utility is relatively high. Risk aversion therefore increases the utility gain of joining the coalition when the player is pivotal and decreases the utility loss of joining the coalition when at least $[c]$ other players join. Participation is thus more appealing for risk averse players; as a result, the equilibrium p is higher.

The rest of this section attempts to organize the data using the following constant relative risk averse (CRRA) utility function:

$$U(y) = \frac{y^{1-\eta}}{1-\eta}$$

where η is the relative risk aversion (RRA) parameter. We have the following maximum likelihood estimates for each treatment, using the procedure introduced by Harrison (2008).

Table 5: Maximum Likelihood Estimations on the RRA Parameter

Treatment	B4	C4	B8	C8
$\hat{\eta}$.26***	.21***	.48***	.53***

Note: This table reports maximum likelihood estimates of relative risk aversion parameter for each treatment. The Newton-Raphson optimization routine is used. *** for $p < 0.001$, ** for $p < 0.01$, and * for $p < 0.05$.

The magnitudes of these risk-aversion estimates do not depart from those that have been found in econometric studies of experimental and field data.^{30,31} According to the risk-aversion classifications in the literature (e.g., Goeree et al. 2003, Holt and Laury 2002), our estimates suggest that subjects in the treatments with $N = 4$ are only slightly risk averse. However, the slight risk-aversion substantially increases efficiency: with $N = 4$, the observed frequencies of participation and coalition formation in the lab are 0.57 and 0.43, significantly larger than the theoretical predictions of 0.444 and 0.234.

We want to emphasize that we do *not* claim risk-aversion is the only reason for the over-

³⁰For instance, the estimations of Holt and Laury (2002) and Campo et al. (2011) lead to RRA coefficients between 0.25 and 0.3; Goeree and Holt (2004) and Goeree et al. (2002) obtain RRA coefficients between 0.46 and 0.56; Goeree et al. (2003) estimate RRA coefficients of approximately 0.43 using their data from matching pennies games and approximately 0.3 using their lottery choice data.

³¹We also obtain the maximum likelihood estimates of η for each session. We find that, there is no significant difference between the estimates for the sessions with and without communication.

participation we observed in our data. In fact, the observed over-participation (especially when $N = 8$) can also be partially explained by the role of social preferences and the conditional participation as discussed in Section 4.1. Unfortunately, our experiment was not designed to separate the role of social preference and the role of risk-aversion and, thus, we are not able to make a precise inference on which aspect is more important. One possible way to make some inference is, however, to see if there is any significant difference in the estimated RRA parameters between $N = 4$ and $N = 8$. It turns out that the estimated RRA parameters for $N = 4$ and $N = 8$ are significantly different from each other (Mann-Whitney U test, $p < 0.005$). If one believes that people's risk-attitude should be the same across different treatments, the observed difference should come from another factor. As discussed in Section 4.1, conditional participation may be the driving force of the higher level of membership and coalition formation in the treatments with $N = 8$ than in the treatments with $N = 4$. However, the above RRA estimation does not take conditional participation into account, and this may lead to the higher estimates of RRA for treatments with $N = 8$. Thus, our analysis on the risk-aversion in this section and the discussion on the conditional participation in Section 4.1 should be viewed as complements rather than substitutes to explain our data.

6 Conclusion

Dixit and Olson (2000) cast doubt on the validity of the Coase Theorem in public goods provision with voluntary participation. They argue that free-riding incentives prevent players from participating sufficiently often in a coalition that aims to provide a lumpy public good, even though the resulting coalition engages in costless and efficient Coasian bargaining. Consequently, the outcome likely falls short of the full efficiency. Based on the influential model of Dixit and Olson (2000), this paper starts with a theoretical analysis of voluntary participation in Coasian bargaining, deriving a few new analytical results. However, the main interest of this paper lies in empirically investigating the extent to which voluntary participation undermines the Coase Theorem through the use of controlled laboratory experiments.

We find that in many cases, as Dixit and Olson predict, the participation level is too low to induce the efficient outcome. However, the observed frequencies of participation and coalition formation are significantly higher than what Dixit and Olson's model suggests, especially when the group size is larger. Risk aversion and conditional participants with social preferences may explain the relatively optimistic outcomes. Pre-play cheap talk communication does not significantly improve participation and efficiency, although it appears to improve coordination.

A few limitations of the paper are worthwhile to be mentioned. First, our experimental design does not allow us to scrutinize whether the social preference or the risk-aversion plays a more important role in the participation decision made by individuals, although we find that the two factors may both contribute to the behavioral patterns we observe. Second, the type of communication we considered in our paper is very limited so that we are silent about what would happen if some other, potentially more realistic, form of communication such as face-to-face communication is implemented. We think these are interesting lines of research and leave them to future research.

Appendix - Derivation of Equation (2) and Proofs

Derivation of Equation (2) Equation (1) could be rewritten as

$$\begin{aligned}
 \frac{(N-1)!}{([\![c]\!] - 1)!(N - [\![c]\!])!} p^{[\![c]\!]-1} (1-p)^{N-[\![c]\!]} &= \sum_{i=[\![c]\!]-1}^{N-1} \frac{(N-1)!}{i!(N-1-i)!} p^i (1-p)^{N-1-i} \frac{c}{1+i} \\
 &\Leftrightarrow \frac{\frac{(N-1)!}{([\![c]\!] - 1)!(N - [\![c]\!])!} p^{[\![c]\!]-1} (1-p)^{N-[\![c]\!]}}{\sum_{i=[\![c]\!]-1}^{N-1} \frac{(N-1)!}{i!(N-1-i)!} p^i (1-p)^{N-1-i} \frac{1}{1+i}} = c \\
 &\Leftrightarrow \frac{\frac{N(N-1)!}{[\![c]\!](\![c]\!] - 1)!(N - [\![c]\!])!} p \cdot p^{[\![c]\!]-1} (1-p)^{N-[\![c]\!]}}{\sum_{i=[\![c]\!]-1}^{N-1} \frac{N(N-1)!}{(i+1)i!(N-1-i)!} p \cdot p^i (1-p)^{N-1-i}} = \frac{c}{[\![c]\!]} \tag{8}
 \end{aligned}$$

Let $i + 1 = j$. Equation (8) becomes Equation (2).

Proof of Proposition 1.

Proof. (i) Let $x = \frac{p}{1-p}$. Define $\sigma(x) \equiv c \int_0^1 (1-t)^{[\![c]\!]-1} (1+tx)^{N-[\![c]\!] } dt$, the left-hand side of (5).

Then

$$\sigma(0) = c \int_0^1 (1-t)^{[\![c]\!]-1} dt < [\![c]\!] \int_0^1 (1-t)^{[\![c]\!]-1} dt = 1$$

where the second equality is obtained by plugging $c = [\![c]\!]$ and $p = 0$ into Equation (5). (Note that, by Remark 2, when c is an integer ($c = [\![c]\!]$), $p = 0$.) Also,

$$\frac{d\sigma}{dx} = c \int_0^1 (1-t)^{[\![c]\!]-1} (N - [\![c]\!]) (1+tx)^{N-[\![c]\!]-1} t dt > 0$$

and

$$\frac{d^2\sigma}{dx^2} = c \int_0^1 (1-t)^{[\![c]\!]-1} (N - [\![c]\!]) (N - [\![c]\!] - 1) (1+tx)^{N-[\![c]\!]-2} t^2 dt \geq 0.$$

Since $\sigma(0) < 1$, and $\sigma(x)$ is increasing and weakly convex in any $x > 0$, there must exist a unique $x > 0$ that satisfies (5). Because of the one-to-one correspondence between p and x , there must exist a unique p that satisfies (5).

(ii) In any interval where $[\![c]\!]$ is a constant, when c increases, according to Equation (5), $\frac{p}{1-p}$ should decrease, implying that p will decrease. \square

Proof of Proposition 2.

Proof. The proposition is evident when $c = 2$ for which $p = 0$ by Remark 2. We focus on the case when $c > 2$. The lower bound of the right-hand side of (2) is $\frac{[\![c]\!]-1}{[\![c]\!]}$. By (4), we can see that the left-hand side of (2) is decreasing in p . Thus if we can show that the left-hand side of (2)

evaluated at $\frac{[c]}{N}$ is less than $\frac{[c]-1}{[c]}$ and thus less than $\frac{c}{[c]}$, then it must be true that $p < \frac{[c]}{N}$. In other words, it is sufficient to show that

$$\frac{\frac{N!}{[c]!(N-[c])!} \left(\frac{[c]}{N}\right)^{[c]} \left(1 - \frac{[c]}{N}\right)^{N-[c]}}{\sum_{j=[c]}^N \frac{N!}{j!(N-j)!} \left(\frac{[c]}{N}\right)^j \left(1 - \frac{[c]}{N}\right)^{N-j}} < \frac{[c]-1}{[c]}$$

which is equivalent to

$$\begin{aligned} \frac{\sum_{j=[c]}^N \frac{N!}{j!(N-j)!} \left(\frac{[c]}{N}\right)^j \left(1 - \frac{[c]}{N}\right)^{N-j}}{\frac{N!}{[c]!(N-[c])!} \left(\frac{[c]}{N}\right)^{[c]} \left(1 - \frac{[c]}{N}\right)^{N-[c]}} &> \frac{[c]}{[c]-1} = 1 + \frac{1}{[c]-1} \\ \Leftrightarrow \frac{\sum_{j=[c]+1}^N \frac{N!}{j!(N-j)!} \left(\frac{[c]}{N}\right)^j \left(1 - \frac{[c]}{N}\right)^{N-j}}{\frac{N!}{[c]!(N-[c])!} \left(\frac{[c]}{N}\right)^{[c]} \left(1 - \frac{[c]}{N}\right)^{N-[c]}} &> \frac{1}{[c]-1}. \end{aligned}$$

It is sufficient to prove that

$$\begin{aligned} \frac{\frac{N!}{([c]+1)!(N-[c]-1)!} \left(\frac{[c]}{N}\right)^{[c]+1} \left(1 - \frac{[c]}{N}\right)^{N-[c]-1}}{\frac{N!}{[c]!(N-[c])!} \left(\frac{[c]}{N}\right)^{[c]} \left(1 - \frac{[c]}{N}\right)^{N-[c]}} &> \frac{1}{[c]-1} \\ \Leftrightarrow \frac{N-[c]}{[c]+1} \frac{\frac{[c]}{N}}{1 - \frac{[c]}{N}} &> \frac{1}{[c]-1} \\ \Leftrightarrow \frac{[c]}{[c]+1} &> \frac{1}{[c]-1}. \end{aligned}$$

By $[c] > 1$, this is equivalent to

$$[c]^2 - 2[c] - 1 > 0.$$

Since $c > 2$, $[c] \geq 3$, so $[c]^2 - 2[c] - 1 > 0$. Hence it is true that when $c > 2$, $p < \frac{[c]}{N}$ and thus $\bar{m} < [c]$. \square

Proof of Lemma 1.

Proof. Replacing $\frac{p}{1-p}$ by x and total differentiating (5) yield

$$\frac{dx}{dN} = -\frac{c \int_0^1 (1-t)^{[c]-1} (1+tx)^{N-[c]} \ln(1+tx) dt}{c \int_0^1 (1-t)^{[c]-1} (N-[c])(1+tx)^{N-[c]-1} t dt} < 0.$$

Since $\frac{dp}{dx} > 0$, we have $\frac{dp}{dN} < 0$. \square

Proof of Proposition 3.

Proof. Substituting $\frac{\bar{m}}{N-\bar{m}} = \frac{p}{1-p}$ into (5) yields

$$\int_0^1 (1-t)^{[c]-1} \left(1 + t \frac{\bar{m}}{N-\bar{m}}\right)^{N-[c]} dt = \frac{1}{c}. \quad (9)$$

Define $H(\bar{m}, N) = \int_0^1 (1-t)^{[c]-1} \left(1 + t \frac{\bar{m}}{N-\bar{m}}\right)^{N-[c]} dt$. Total differentiation of (9) yields

$$\frac{d\bar{m}}{dN} = -\frac{\frac{\partial H}{\partial N}}{\frac{\partial H}{\partial \bar{m}}} \quad (10)$$

where

$$\frac{\partial H}{\partial \bar{m}} = \int_0^1 (1-t)^{[c]-1} (N-[c]) \left(1 + t \frac{\bar{m}}{N-\bar{m}}\right)^{N-[c]-1} t \frac{N}{(N-\bar{m})^2} dt > 0$$

and

$$\begin{aligned} \frac{\partial H}{\partial N} &= \int_0^1 (1-t)^{[c]-1} \left[\begin{array}{l} \left(1 + t \frac{\bar{m}}{N-\bar{m}}\right)^{N-[c]} \ln\left(1 + t \frac{\bar{m}}{N-\bar{m}}\right) \\ - (N-[c]) \left(1 + t \frac{\bar{m}}{N-\bar{m}}\right)^{N-[c]-1} t \frac{\bar{m}}{(N-\bar{m})^2} \end{array} \right] dt \\ &= \int_0^1 (1-t)^{[c]-1} \left(1 + t \frac{\bar{m}}{N-\bar{m}}\right)^{N-[c]-1} \left[\left(1 + t \frac{\bar{m}}{N-\bar{m}}\right) \ln\left(1 + t \frac{\bar{m}}{N-\bar{m}}\right) - \frac{(N-[c])t\bar{m}}{(N-\bar{m})^2} \right] dt \\ &= \int_0^1 (1-t)^{[c]-1} \left(1 + t \frac{\bar{m}}{N-\bar{m}}\right)^{N-[c]-1} A(t) dt, \end{aligned}$$

where

$$A(t) = \left(1 + t \frac{\bar{m}}{N-\bar{m}}\right) \ln\left(1 + t \frac{\bar{m}}{N-\bar{m}}\right) - (N-[c])t \frac{\bar{m}}{(N-\bar{m})^2}.$$

It is easy to see that $A(0) = 0$, and

$$\begin{aligned} A'(t) &= \frac{\bar{m}}{N-\bar{m}} \ln\left(1 + t \frac{\bar{m}}{N-\bar{m}}\right) + \frac{1 + t \frac{\bar{m}}{N-\bar{m}}}{1 + t \frac{\bar{m}}{N-\bar{m}}} \frac{\bar{m}}{N-\bar{m}} - (N-[c]) \frac{\bar{m}}{(N-\bar{m})^2} \\ &= \frac{\bar{m}}{N-\bar{m}} \left[\ln\left(1 + t \frac{\bar{m}}{N-\bar{m}}\right) + 1 - \frac{N-[c]}{N-\bar{m}} \right] \\ &= \frac{\bar{m}}{N-\bar{m}} \left[\ln\left(1 + t \frac{\bar{m}}{N-\bar{m}}\right) + \frac{[c]-\bar{m}}{N-\bar{m}} \right] \\ &> 0 \end{aligned}$$

for any $0 < t < 1$, where the inequality comes from the fact that $\bar{m} < [c]$ when $c \geq 2$ (Proposition 2). Therefore, $A(t) > 0$ for any $0 < t < 1$. We thus have $\frac{\partial H}{\partial N} > 0$ as well. So eventually we have

$$\frac{d\bar{m}}{dN} = -\frac{\frac{\partial H}{\partial N}}{\frac{\partial H}{\partial \bar{m}}} < 0.$$

□

Proof of Remark 3.

Proof. With $x = \frac{p}{1-p}$, for $0 < c < 1$, equilibrium condition (5) becomes

$$c \int_0^1 (1+tx)^{N-1} dt = 1,$$

which can be rewritten as

$$(1+x)^N - \frac{Nx}{c} - 1 = 0.$$

Total differentiation yields

$$\frac{dx}{dN} = -\frac{(1+x)^N \ln(1+x) - \frac{x}{c}}{N(1+x)^{N-1} - \frac{N}{c}}.$$

Taking the derivative of the probability that the public good is not provided by the coalition, $(1-p)^N$, with respect to N yields

$$\begin{aligned} \frac{d(1-p)^N}{dN} &= (1-p)^{N-1} \left[-N \frac{dp}{dx} \frac{dx}{dN} + (1-p) \ln(1-p) \right] \\ &= (1-p)^{N-1} \left[N \frac{1}{(1+x)^2} \frac{(1+x)^N \ln(1+x) - \frac{x}{c}}{N(1+x)^{N-1} - \frac{N}{c}} + \frac{1}{1+x} \ln\left(\frac{1}{1+x}\right) \right] \\ &= \frac{(1-p)^{N-1}}{1+x} \left[\frac{(1+x)^N \ln(1+x) - \frac{x}{c}}{(1+x)^N - \frac{1+x}{c}} - \ln(1+x) \right] \\ &= \frac{(1-p)^{N-1}}{1+x} \frac{\frac{(1+x)\ln(1+x)}{c} - \frac{x}{c}}{(1+x)^N - \frac{1+x}{c}}. \end{aligned}$$

The first term of the last equality is positive. To show that $\frac{d(1-p)^N}{dN} > 0$, we want to show that both the numerator and the denominator of the second term in the last equality are positive. We first show that $x > 0$. This is because the equilibrium condition at the beginning of this proof is not satisfied if $x = 0$. Then define $F(x) = \frac{1}{c}[(1+x)\ln(1+x) - x]$, i.e., the numerator. By $F'(x) = \frac{\ln(1+x)}{c} > 0$ for any $x > 0$ and $F(0) = 0$, we know that the numerator $F(x) > 0$ for any

$x > 0$. We then show that the denominator is positive. Suppose not. We have

$$\begin{aligned} (1+x)^N - \frac{1+x}{c} &\leq 0 \Rightarrow \\ c(1+x)^{N-1} &\leq 1 \Rightarrow \\ c \int_0^1 (1+x)^{N-1} dt &\leq 1 \Rightarrow \\ c \int_0^1 (1+tx)^{N-1} dt &< 1, \end{aligned}$$

where the last inequality comes from $0 < t < 1$ and $x > 0$. The last inequality contradicts the equilibrium condition at the beginning of this proof. Thus we have $\frac{d(1-p)^N}{dN} > 0$. Hence, $G_1(N) = 1 - (1-p)^N$ is decreasing in N . \square

Proof of Proposition 5.

Proof. Step 1: Define $g(p) = \frac{(N-1)!}{([\![c]\!] - 1)!(N - [\![c]\!]!) p^{[\![c]\!] - 1} (1-p)^{N - [\![c]\!]}$ $\left(1 - \frac{c}{[\![c]\!]}\right)$, the left-hand side of (1), and $G(p) = c \sum_{i=[\![c]\!]}^{N-1} \frac{(N-1)!}{i!(N-1-i)!} p^i (1-p)^{N-1-i} \frac{1}{1+i}$, the right-hand side of (1). We establish the following claims.

Claim 1: There exists a unique intersection between $g(p)$ and $G(p)$ in the interval of $0 < p < 1$. This is proved by Proposition 1.

Claim 2: $g(0) = G(0) = 0$. This is straightforward.

Claim 3: $g(p)$ is hump-shaped, first increasing and then decreasing in p . To prove this, we take the derivative of g with respect to p :

$$g'(p) = \frac{(N-1)!}{([\![c]\!] - 1)!(N - [\![c]\!]!) \left(1 - \frac{c}{[\![c]\!]}\right)} p^{[\![c]\!] - 1} (1-p)^{N - [\![c]\!] - 1} \left[\frac{[\![c]\!] - 1}{p} - \frac{N - [\![c]\!] - 1}{1-p} \right].$$

Thus when $p < \frac{[\![c]\!] - 1}{N - 1}$, $\frac{dg(p)}{dp} > 0$, and when $p > \frac{[\![c]\!] - 1}{N - 1}$, $\frac{dg(p)}{dp} < 0$. So $g(p)$ is single-peaked, first increasing and then decreasing in p .

Claim 4: $G'(p) > 0$ when $g'(p) > 0$. To see this, take derivative of G with respect to p :

$$G'(p) = c \sum_{i=[\![c]\!]}^{N-1} \frac{(N-1)!}{i!(N-1-i)!} \frac{1}{1+i} p^i (1-p)^{N-1-i} \left[\frac{i}{p} - \frac{N-1-i}{1-p} \right].$$

When $g'(p) > 0$, $\frac{[\![c]\!] - 1}{p} - \frac{N - [\![c]\!] - 1}{1-p} > 0$; then it must be true that $\frac{i}{p} - \frac{N-1-i}{1-p} > 0$ for $i > [\![c]\!] - 1$. So $G'(p) > 0$ when $g'(p) > 0$.

Claim 5: When p converges to 0 from the right, $G(p) < g(p)$. The reason is that,

$$\frac{G(p)}{g(p)} = \sum_{i=\lceil c \rceil}^{N-1} \left[\frac{\frac{(N-1)!}{i!(N-1-i)!} \frac{c}{1+i}}{\frac{(N-1)!}{(\lceil c \rceil - 1)!(N - \lceil c \rceil)!} 1 - \frac{c}{\lceil c \rceil}} \left(\frac{p}{1-p} \right)^{i - \lceil c \rceil + 1} \right].$$

When p converges to 0 from the right, $\left(\frac{p}{1-p}\right)^{i - \lceil c \rceil + 1}$ converges to 0. $G(p)$ contains finite terms, the ratio of each of which to $g(p)$ is 0. So it must be true that $G(p) < g(p)$ when p converges to 0.

Step 2: The equilibrium condition under risk-aversion is

$$\begin{aligned} & \frac{(N-1)!}{(\lceil c \rceil - 1)!(N - \lceil c \rceil)!} p^{\lceil c \rceil - 1} (1-p)^{N - \lceil c \rceil} \left[U\left(1 - \frac{c}{\lceil c \rceil}\right) - U(0) \right] \\ &= \sum_{i=\lceil c \rceil}^{N-1} \frac{(N-1)!}{i!(N-1-i)!} p^i (1-p)^{N-1-i} \left[U(1) - U\left(1 - \frac{c}{1+i}\right) \right]. \end{aligned} \quad (11)$$

Given any utility function U , we can re-scale U so that

$$U\left(1 - \frac{c}{\lceil c \rceil}\right) - U(0) = 1 - \frac{c}{\lceil c \rceil} \quad (12)$$

without changing preferences. Using (12), the left-hand side of (11) is the same as the left-hand side of the equilibrium condition under risk neutrality (1), i.e. $g(p)$. For $i \geq \lceil c \rceil$, we have

$$1 > 1 - \frac{c}{1+i} \geq 1 - \frac{c}{\lceil c \rceil} > 0. \quad (13)$$

Using (12) and the assumption that U is strictly concave in some interval within $\left[1 - \frac{c}{\lceil c \rceil}, 1\right]$, inequality (13) implies

$$U(1) - U\left(1 - \frac{c}{1+i}\right) < \frac{c}{1+i}.$$

So, the right-hand side of (11) is less than the right-hand side of (1), the equilibrium condition under risk neutrality, i.e. $G(p)$. With risk-aversion, the left-hand side of the equilibrium condition does not change and the right-hand side shifts downward. Taking into account the claims about the shapes of $g(p)$ and $G(p)$ in step 1, the equilibrium probability must be higher under risk-aversion. \square

References

- [1] Aumann, Robert. 1974. "Subjectivity and Correlation in Randomized Strategies." *Journal of Mathematical Economics*, 1, 67-96.
- [2] Bagnoli, Mark and Barton Lipman. 1988. "Successful Takeovers without Exclusion." *Review of Financial Studies*, 1(1), 89-110.
- [3] Blanco, Mariana, Dirk Engelmann, Alexander K. Koch, and Hans-Theo Normann. 2010. "Belief Elicitation in Experiments: Is There a Hedging Problem?" *Experimental Economics*, 13: 412-438.
- [4] Bochet, Olivier, Talbot Page and Louis Putterman, 2006. "Communication and Punishment in Voluntary Contribution Experiments." *Journal of Economic Behavior & Organization*, 60: 11-26.
- [5] Bolton, Gary and Kalyan Chatterjee. 1996. "Coalition Formation, Communication, and Coordination: An Exploratory Experiment." In: Zeckhauser, R., Keeney, R., Sebenius, J. (Eds.), *Wise Choices: Decisions, Games, and Negotiations*, Harvard Business School Press, 253-268.
- [6] Burger, Nicholas and Charles Kolstad. 2010. "International Environmental Agreements: Theory Meets Experimental Evidence." Working Paper.
- [7] Campo, Sandra, Emmanuel Guerre, Isabelle Perrigne and Quang Vuong. 2011. "Semiparametric Estimation of First-Price Auctions with Risk-Averse Bidders." *Review of Economic Studies*, 78(1), 112-147.
- [8] Cason, Timothy, Tatsuyoshi Saijo and Takehiko Yamato. 2002. "Voluntary Participation and Spite in Public Good Provision Experiments: An International Comparison." *Experimental Economics*, 5, 133-153.
- [9] Cason, Timothy, Tatsuyoshi Saijo, Takehiko Yamato and Konomu Yokotani. 2004. "Non-excludable Public Good Experiments." *Games and Economic Behavior*, 49, 81-102.
- [10] Chaudhuri, Ananish, Sara Graziano, and Pushkar Maitra. 2006. "Social Learning and Norms in a Public Goods Experiment with Intergenerational Advice." *Review of Economic Studies*, 73(2), 357-380.
- [11] Chaudhuri, Ananish. 2011. "Sustaining Cooperation in Laboratory Public Goods Experiments: A Selective Survey of the Literature." *Experimental Economics*, 14: 47-83.

- [12] Coase, Ronald. 1960. "The Problem of Social Cost." *Journal of Law and Economics*, 3(1), 1-44.
- [13] Crawford, Vincent. 1990. "Explicit Communication and Bargaining Outcome." *American Economic Review*, Papers and Proceedings of the Hundred and Second Annual Meeting of the American Economic Association, 80(2): 213-219.
- [14] Crawford, Vincent. 1998. "A Survey of Experiments on Communication via Cheap Talk." *Journal of Economic Theory*, 78(2), 286-298.
- [15] Dannenberg, Astrid, Andreas Lange and Bodo Sturm. 2010. "On the Formation of Coalitions to Provide Public Goods - Experimental Evidence from the Lab." NBER Working Paper 15967.
- [16] Dixit, Avinash and Mancur Olson. 2000. "Does Voluntary Participation Undermine the Coase Theorem?" *Journal of Public Economics*, 76(3), 309-335.
- [17] Farrell, Joseph. 1987. "Cheap Talk, Coordination, and Entry." *Rand Journal of Economics*, 18(1), 34-39.
- [18] Farrell, Joseph and Matthew Rabin. 1996. "Cheap Talk." *Journal of Economic Perspectives*, 10(3), 103-118.
- [19] Fehr, Ernst and Simon Gächter. 2000. "Fairness and Retaliation: The Economics of Reciprocity." *Journal of Economic Perspectives*, 14(3), 159-181.
- [20] Fischbacher, Urs, Simon Gächter, and Ernst Fehr. 2001. "Are People Conditionally Cooperative? Evidence from a Public Goods Experiment." *Economics Letters*, 71(3), 397-404.
- [21] Fischbacher, Urs. 2007. "z-Tree: Zurich Toolbox for Ready-made Economic Experiments." *Experimental Economics*, 10(2), 171-178.
- [22] Frey, Bruno, and Stephan Meier. 2004. "Social Comparisons and Pro-Social Behavior: Testing 'Conditional Cooperation' in a Field Experiment." *American Economic Review*, 94(5), 1717-1722.
- [23] Goeree, Jacob and Charles Holt. 2004. "A Model of Noisy Introspection." *Games and Economic Behavior*, 46(2), 365-382.
- [24] Goeree, Jacob, Charles Holt and Thomas Palfrey. 2002. "Quantal Response Equilibrium and Overbidding in Private-Value Auctions." *Journal of Economic Theory*, 104, 247-272.
- [25] Goeree, Jacob, Charles Holt and Thomas Palfrey. 2003. "Risk Averse Behavior in Generalized Matching Pennies Games." *Games and Economic Behavior*, 45(1), 97-113.

- [26] Harrington, Joseph. 2001. "A Simple Game-Theoretic Explanation for the Relationship between Group Size and Helping." *Journal of Mathematical Psychology*, 45, 389-392.
- [27] Harrison, Glenn. 2008. "Maximum Likelihood Estimation of Utility Functions Using Stata." Working Paper.
- [28] Harrison, Glenn and Elisabet Rutström. 2008. "Risk-aversion in the Laboratory." In Cox, JC and GW Harrison (eds.), *Risk-aversion in Experiments*, Bingley, UK: Emerald, Research in Experimental Economics, Volume 12.
- [29] Heijnen, Pim. 2009. "On the Probability of Breakdown in Participation Games." *Social Choice and Welfare*, 32, 493-511.
- [30] Holmstrom, Bengt and Barry Nalebuff. 1992. "To the Raider Goes the Surplus? A Reexamination of the Free-rider Problem." *Journal of Economics and Management Strategy*, 1, 37-62.
- [31] Holt, Charles and Susan Laury. 2002. "Risk Aversion and Incentive Effects." *American Economic Review*, 92(5), 1644-1655.
- [32] Hong, Fuhai and Larry Karp. 2012. "International Environmental Agreements with Mixed Strategies and Investment," *Journal of Public Economics*, 96(9-10), 685-697.
- [33] Hong, Fuhai and Larry Karp. 2014. "International Environmental Agreements with Endogenous or Exogenous Risk," *Journal of the Association of Environmental and Resource Economists*, 1(3), 365-394.
- [34] Isaac, Mark and James Walker. 1988a. "Group Size Effects in Public Goods Provision: The Voluntary Contribution Mechanism." *Quarterly Journal of Economics*, 103, 179-200.
- [35] Isaac, Mark and James Walker. 1988b. "Communication and Free-riding Behavior: The Voluntary Contributions Mechanism." *Economic Inquiry*, 26, 585-608.
- [36] Isaac, Mark, James Walker and Arlington Williams. 1994. "Group Size and the Voluntary Provision of Public Goods: Experimental Evidence Utilizing Large Groups." *Journal of Public Economics*, 54, 1-36.
- [37] Kosfeld, Michael, Akira Okada and Arno Riedl. 2009. "Institution Formation in Public Goods Games." *American Economic Review*, 99(4), 1335-1355.
- [38] Ledyard, John. 1995. "Public goods: A Survey of Experimental Research." In Kagel, JH and AE Roth (eds.), *The Handbook of Experimental Economics*, Princeton University Press.

- [39] McEvoy, David and Todd Cherry. 2010. "Endogenous Minimum Participation in International Environmental Agreements: Experimental Evidence." Working Paper.
- [40] Myerson, Roger. 1986. "Multistage Games with Communication." *Econometrica*, 54(2), 323-358.
- [41] Olson, Mancur. 1965. *The Logic of Collective Action: Public Goods and the Theory of Groups*, Harvard University Press.
- [42] Orbell, John, Robyn Dawes and Alphons van de Kragt. 1990. "The Limits of Multilateral Promising." *Ethics*, 100(3), 616-627.
- [43] Palfrey, Thomas and Howard Rosenthal. 1984. "Participation and the Provision of Discrete Public Goods: A Strategic Analysis." *Journal of Public Economics*, 24, 171-193.
- [44] Palfrey, Thomas and Howard Rosenthal. 1991. "Testing for Effects of Cheap Talk in a Public Goods Game with Private Information." *Games and Economic Behavior*, 3, 183-220.
- [45] Rapoport, Amnon. 1999. "Provision of Binary Public Goods in Single-Period Social Dilemmas: Theories and Experiments." Working Paper.
- [46] Saijo, Tatsuyoshi and Takehiko Yamato. 1999. "A Voluntary Participation Game with a Non-excludable Public Good." *Journal of Economic Theory*, 84(2), 227-242.
- [47] Saijo, Tatsuyoshi and Takehiko Yamato. 2010. "Fundamental Impossibility Theorems on Voluntary Participation in the Provision of Non-excludable Public Goods." *Review of Economic Design*, 14(1-2), 51-73.
- [48] Shinohara, Ryusuke. 2009. "The Possibility of Efficient Provision of a Public Good in Voluntary Participation Games." *Social Choice and Welfare*, 32, 367-387.
- [49] Snedecor, GW and WG Cochran. 1989. *Statistical Methods*, Iowa State University Press.
- [50] Taylor, Michael, 1987. "The Possibility of Cooperation." *Cambridge University Press*, Cambridge.
- [51] Teyssier, Sabrina. 2012. "Inequity and Risk-aversion in Sequential Public Good Games." *Public Choice*, 151(1-2), 91-119.
- [52] Wang, Renguan. 1994. *Introduction to Probability*, Peking University Press, Beijing.
- [53] Zelmer, Jennifer. 2003. "Linear Public Goods Experiments: A Meta-Analysis." *Experimental Economics*, 6, 299-310.

Online Appendix to “Voluntary Participation in Public Goods Provision with Coasian Bargaining” by Fuhai Hong and Wooyoung Lim

A. Alternative Derivation of Equation (5)

Wang (1994) shows the following mathematical identity.

$$\sum_{i=k}^n \frac{n!}{i!(n-i)!} p^i (1-p)^{n-i} \equiv \frac{n!}{(k-1)!(n-k)!} \int_0^p t^{k-1} (1-t)^{n-k} dt. \quad (14)$$

Define

$$\mu \equiv \frac{\sum_{j=[c]}^N \frac{N!}{j!(N-j)!} p^j (1-p)^{N-j}}{\frac{N!}{[c]!(N-[c])!} p^{[c]} (1-p)^{N-[c]}}.$$

Therefore, Equation (2) becomes

$$\frac{1}{\mu} = \frac{c}{[c]}. \quad (15)$$

Using Equation (14), we have

$$\begin{aligned} \mu &= \frac{\frac{N!}{([c]-1)!(N-[c])!} \int_0^p t^{[c]-1} (1-t)^{N-[c]} dt}{\frac{N!}{[c]!(N-[c])!} p^{[c]} (1-p)^{N-[c]}} \\ &= \frac{[c] \int_0^p t^{[c]-1} (1-t)^{N-[c]} dt}{p^{[c]} (1-p)^{N-[c]}}. \end{aligned} \quad (16)$$

Note that

$$\begin{aligned} &\int_0^1 (1-t)^{[c]-1} \left(1 + t \frac{p}{1-p}\right)^{N-[c]} dt \\ &= \int_0^1 (1-t)^{[c]-1} \left(1 - t + \frac{t}{1-p}\right)^{N-[c]} dt. \end{aligned}$$

Let $s = 1 - t$, then

$$\begin{aligned}
& \int_0^1 (1-t)^{[c]-1} \left(1-t + \frac{t}{1-p}\right)^{N-[c]} dt \\
&= \int_0^1 s^{[c]-1} \left(s + \frac{1-s}{1-p}\right)^{N-[c]} ds \\
&= \int_0^1 s^{[c]-1} \left(\frac{1}{1-p} - \frac{p}{1-p}s\right)^{N-[c]} ds \\
&= \left(\frac{1}{1-p}\right)^{N-[c]} \int_0^1 s^{[c]-1} (1-ps)^{N-[c]} ds.
\end{aligned}$$

Let $w = ps$, then

$$\begin{aligned}
& \left(\frac{1}{1-p}\right)^{N-[c]} \int_0^1 s^{[c]-1} (1-ps)^{N-[c]} ds \\
&= \left(\frac{1}{1-p}\right)^{N-[c]} \int_0^p \left(\frac{w}{p}\right)^{[c]-1} (1-w)^{N-[c]} \frac{1}{p} dw \\
&= \left(\frac{1}{1-p}\right)^{N-[c]} \left(\frac{1}{p}\right)^{[c]} \int_0^p w^{[c]-1} (1-w)^{N-[c]} dw.
\end{aligned}$$

So

$$\int_0^p w^{[c]-1} (1-w)^{N-[c]} dw = (1-p)^{N-[c]} p^{[c]} \int_0^1 (1-t)^{[c]-1} \left(1+t\frac{p}{1-p}\right)^{N-[c]} dt. \quad (17)$$

Using this, Equation (16) becomes

$$\begin{aligned}
\mu &= \frac{[c] \int_0^p w^{[c]-1} (1-w)^{N-[c]} dw}{p^{[c]} (1-p)^{N-[c]}} \\
&= \frac{[c] (1-p)^{N-[c]} p^{[c]} \int_0^1 (1-t)^{[c]-1} \left(1+t\frac{p}{1-p}\right)^{N-[c]} dt}{p^{[c]} (1-p)^{N-[c]}} \\
&= [c] \int_0^1 (1-t)^{[c]-1} \left(1+t\frac{p}{1-p}\right)^{N-[c]} dt,
\end{aligned} \quad (18)$$

where in the first equality, we just replace t by w , and in the second equality, we make use of Equation (17). Substituting Equation (18) into Equation (15), we obtain Equation (5).

B. Translated Experimental Instructions for Treatment C8

Welcome to the experiment. This experiment studies decision-making among individuals. In the next two hours or less, you will participate in 30 rounds of decision-making. Please carefully read the instructions below; the cash payment you will receive at the end of the experiment depends on how well you make your decisions in accordance with the rules described in these instructions. At the end of today's session, you will be paid privately and in cash for your decisions.

You will never be asked to reveal your identity to anyone during the course of the experiment. Your name will never be associated with any of your decisions. To keep your decisions private, please do not reveal your choices to any other participant in the experiment.

Your Tasks

In each round of the experiment, you will form a group of eight with seven other randomly matched individuals in this room. You will be asked to make two decisions in the following two stages:

1. **Communication Stage:** At the beginning of each round, the computer will ask if you want to send a message to group members saying that you are going to participate in **Coalition C** by clicking "Yes" or "No". Your "Yes" or "No" choice here does not directly affect your payoff. Subsequently, the total number of people in your group who choose "Yes" will be announced to everyone in your group.
2. **Participation Stage:** After learning the total number of "Yes" messages in your group, you will be asked to decide whether to be a member of **Coalition C**. You are free to choose to (or not to) participate in Coalition C, regardless of your answer to the previous question. After all of the individuals in your group have made their decisions, the total number of members, denoted by N , of Coalition C will be publicly announced in the group.

The amount of money that you earn depends on your own participation decision and the participation decisions of the other seven people in your group during the second (participation) stage.

Your Earnings for Each Round

Your earnings for each round will be determined as follows:

1. If the total number of members in **Coalition C**, N , is strictly greater than two, then the coalition is formed and

- Everyone in your group will receive 100 points.
 - The members of **Coalition C** will pay $\frac{250}{N}$.
2. If the total number of members in **Coalition C**, N , is less than or equal to two, then Coalition C is not formed and
- No one in your group will receive 100 points.
 - No one will pay anything.

For example,

- If you choose to be a member of Coalition C and the total number of members of Coalition C is 5, then your earnings for the round are $100 - (\frac{250}{5}) = 50$.
- If you choose not to be the a member of Coalition C and the total number of members of Coalition C is 5, then your earnings for the round are $100 - 0 = 100$.
- If the total number of members of Coalition C is 2, then regardless of your decision, you receive nothing and there is no need to pay anything. Thus, your earnings for the round are 0.

The following table summarizes how your earnings in each round depend on the decisions made by you and the other people in your group:

Membership of Coalition C (excluding you)	0	1	2	3	4	5	6	7
If you choose to be a member of Coalition C	0	0	16	37	50	58	64	68
If you choose not to be a member of Coalition C	0	0	0	100	100	100	100	100

Table 6: Summary of your possible earning in each round

The Rundown of the Experiment

1. At the beginning of each round, the computer will randomly group you with 7 other individuals.
2. The computer will ask “Do you want to send a message to your group members saying that you are going to participate in Coalition C?” Each individual answers the question by clicking “Yes” or “No”.
3. The total number of people in your group who choose “Yes” will be announced to everyone in your group.

4. The computer will ask “How many people (excluding yourself) in your group do you believe will participate in Coalition C?” You are free to choose any integer between 0 and 7. This choice does not affect your earnings in the round and will never be revealed to other people in the room.
5. Each individual makes his/her decision whether to be a member of Coalition C.
6. The total number of members of Coalition C is announced.
7. Payoff will be computed accordingly.

In all but the final (30th) rounds, the above steps will be repeated once the round is over. The completion of the 30th round entails the end of the experiment. The computer randomly selects three rounds for your payment. Your total payment will be the sum of the earnings you received in the selected rounds divided by 5 plus 10 RMB yuan for taking part in the experiment.

Remember that you have to make your decisions entirely on your own; *please do not discuss your decisions with any other individuals in the experiment.*

Adminstration

You input your decisions with the mouse in front of you. Your decisions and your monetary payment will be kept confidential. Upon finishing the experiment, you will receive your payment. You will be asked to sign your name to acknowledge your receipt of the payment (which will not be used for tax purposes). You are then free to leave. You may start now. Good luck!

C. Original Experimental Instructions for Treatment C8

说明

欢迎参与本次实验。这一实验研究个体间的决策。在接下来的两个小时或更短的时间里，你将参加 30 个回合的决策制定。请认真阅读以下说明；实验结束后你得到的现金收入将取决于你如何根据这些说明制定你的决策。在这今天的这组实验结束时，我们将以不公开的方式以现金支付你的决策。

在实验过程中，你不会被要求公开你的身份。你的名字不会你的决策出现在一起。为了保持你的决策的私密性，请不要将你的选择告诉任何其他实验参与者。

你的任务

在本次实验的每一个回合中，你都将与这个房间里被随机选定的其他七人组成一个八人的小组。你将根据以下两个阶段作出两个决策：

1. 交流阶段。在每个回合的开始，电脑将问你是否愿意给你的小组成员发一个讯息说你参加到**联盟 C**中。你将通过点击“是”或者“否”来回答。你选择“是”或者“否”并不会直接影响你的所得。然后，你所在的小组中回答“是”的人数将在小组中公布。
2. 参与阶段。在看到你在所在的小组中选择“是”的人数之后，你将决定是否成为**联盟 C**的成员。不论你如何回答之前的问题，你都可以自由地选择参加（或者不参加）到联盟 C 中。在你所在的组中所有人都作出决定之后，联盟 C 的总的成员数，以 N 表示，将在小组中公布。

你所获得的钱数取决于在第二个（参与）阶段，你自己的参与决定和你所在的小组中其它七人的参与决定。

每一回合中你的所得

在每一回合中你的所得将由以下方式决定：

1. 如果**联盟 C**中的总的成员数， N ，严格大于 2，那么联盟 C 得以形成，
 - 你所在小组中每人将获得 100。
 - 只有**联盟 C**中的成员需要支付 $250/N$ 。
2. 如果**联盟 C**中的总的成员数， N ，小于或者等于 2，那么联盟 C 无法形成，
 - 你所在小组中任何人都无法获得 100。
 - 任何人都无需支付任何费用。

例如，

- 如果你选择成为联盟 C 的一员，而联盟 C 的总成员数是 5，那么在这一回合你的所得是 $100 - (250/5) = 50$ 。
- 如果你没有选择成为联盟 C 的一员，而联盟 C 的总成员数是 5，那么在这一回合你的所得是 $100 - 0 = 100$ 。
- 如果联盟 C 的总成员数是 2，那么无论你的决定是什么，你既得不到任何收入也不需要支付任何成本。所以，你的所得是 0。

下表概括了在每一回合中，你的所得如何取决于你和组内他人的决策

联盟 C 的成员数（除你以外）	0	1	2	3	4	5	6	7
如果你选择成为联盟 C 的成员	0	0	16	37	50	58	64	68
如果你选择不成为联盟 C 的成员	0	0	0	100	100	100	100	100

表 1: 在每一回合中，你的可能的所得

实验的顺序

1. 在每一回合的开始，电脑将随机地将你与其他 7 人编成一个小组。
2. 电脑将会问：“你是否愿意给你的组员发一个讯息说你参加联盟 C？”每个人通过点击“是”或者“否”来回答这个问题。
3. 你所在的小组中选择“是”的人数将在小组中公布。
4. 电脑将会问：“你相信在你所在的小组中(除你以外)有多少人将参加到联盟 C 中来？”你可以从 0 到 7 中任选一个整数。这一选择并不影响你在回合中的收入，也不会被透露给房间里的任何其他人。
5. 每个人作出他/她的决定：是否成为联盟 C 的成员。
6. 联盟 C 的总的成员数被公布。
7. 所得随之被确定。

在除了最后一个回合（即第 30 个回合）以外的所有回合里，每当一个回合结束，以上步骤将被重复。第 30 个回合完成后整个实验将结束。电脑将随机选择三个回合来支付你的所得。你的总收入将是你在被选的回合并中的所得之和除以 5 再加上 10 元人民币的参与费。

请记住你必须完全依靠自己作出决定；请不要与实验中的任何其他人讨论你的决定。

管理

你使用你面前的鼠标输入你的决定。你的决策和你的货币所得将被保密。在完成实验之后，你就会得到你的收入。我们将请你签名确认你的收入（你的签名不会被用于税收目的），然后你就可以离开了。现在你可以开始了。祝你好运！

D. Supplementary Regression Tables

Table 7: Logit Regression Results on Participation: $N = 4$

	(1)	(2)	(3)	(4)	(5)	(6)
Communicate	0.8534 (0.1370)	0.6914 (0.2003)	0.8533 (0.0896)	0.8225 (0.1362)	0.6455 (0.1952)	0.8215 (0.1062)
Pivot	2.345*** (0.3104)	2.333*** (0.3108)	2.338*** (0.1653)			
Belief				1.465** (0.2168)	1.425* (0.1994)	1.437*** (0.1000)
Superfluous				0.2204*** (0.0548)	0.2233*** (0.0547)	0.2224*** (0.0271)
Superfluous \times Belief				Omitted	Omitted	Omitted
Session Effects Controlled for	No	Yes	No	No	Yes	No
Session-level RE Model	No	No	Yes	No	No	Yes
Pseudo R^2	0.0366	0.0394	N.A.	0.0505	0.0546	N.A.
No. of Observations	3600	3600	3600	3600	3600	3600

Note: Using data from treatments B4 and C4, this table reports odds ratios from logit regressions on participation, controlling for period effects. Standard errors are reported in parentheses. In Columns (1), (2), (4) and (5), standard errors are clustered at the individual level. Columns (2) and (5) control for session effects, while Columns (3) and (6) estimate the random effect model at the session level. The interaction term *Superfluous* \times *Belief* is dropped because with $N = 4$, when *Superfluous*=1, *Belief* can only take one value, which is 3. *** for $p < 0.001$, ** for $p < 0.01$, and * for $p < 0.05$.

Table 8: Logit Regression Results on Participation: $N = 8$

	(1)	(2)	(3)	(4)	(5)	(6)
Communicate	1.083 (0.1879)	1.184 (0.3500)	1.083 (0.0842)	1.082 (0.1885)	1.218 (0.3645)	1.082 (0.1013)
Pivot	2.164*** (0.2601)	2.183*** (0.2621)	2.169*** (0.1539)			
Belief				1.403** (0.1668)	1.388** (0.1543)	1.396*** (0.1046)
Superfluous				0.3513* (0.1565)	0.3239** (0.1423)	0.3378*** (0.0782)
Superfluous \times Belief				0.9010 (0.1451)	0.9230 (0.1428)	0.9116 (0.0840)
Session Effects Controlled for	No	Yes	No	No	Yes	No
Session-level RE Model	No	No	Yes	No	No	Yes
Pseudo R^2	0.0291	0.0306	N.A.	0.0398	0.0419	N.A.
No. of Observations	4320	4320	4320	4320	4320	4320

Note: Using data from treatments B8 and C8, this table reports odds ratios from logit regressions on participation, controlling for period effects. Standard errors are reported in parentheses. In Columns (1), (2), (4) and (5), standard errors are clustered at the individual level. Columns (2) and (5) control for session effects, while Columns (3) and (6) estimate the random effect model at the session level. *** for $p < 0.001$, ** for $p < 0.01$, and * for $p < 0.05$.

Table 9: Coalition Formation: Communication Treatments

	Total	$N = 4$	$N = 8$
	(1)	(2)	(3)
$N8$	2.100*** (0.0267)		
$N_y = 3$	1.398* (0.2290)	1.546*** (0.1647)	1.205 (0.6211)
Pseudo R^2	0.0570	0.0559	0.0988
No. of observations	720	450	270

Note: Using group level data, this table reports odds ratios from logit regressions on coalition formation, controlling for period effects and session effects. Standard errors clustered at the session level are in parentheses. *** for $p < 0.001$, ** for $p < 0.01$, and * for $p < 0.05$.